

Carrier dynamic and carrier Temperature in Quantum Well

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Abstract— Carrier temperature in quantum well (QW) semiconductor optical amplifier (SOA) has been studied, depending density matrix theory (DMT) and carrier dynamic in quantum well. The effect of volume carrier density, doping on effect, pulse shape, nonradiative relaxation, and nonlinear gain coefficients on the carrier temperature and carrier heating has been investigated. The theoretical results show that; the time recovery increases straightforward with nonradiative recombination, where the relaxation time of nonradiative recombination reduces the rate of carrier occupation in quantum states. Also, the carrier temperature increases with carrier density, free carrier absorption, carrier heating lifetime and reduces with pulse shape of pump pulses.

Keywords: semiconductor optical amplifier, quantum well, Carrier temperature.

I. INTRODUCTION

Semiconductor optical amplifier (SOA) is a very interesting device; it has taken a great attention in a past few years of science. Due to nonlinearity, SOA's has been used as a technique for wavelength division multiplexing networks [1,2]. Carrier density pulsation (CDP), spectral hole burning (SHB) and carrier heating (CH) are physical mechanisms contributing toward wavelength conversion. CDP is an interband process, it results from the beating between the pump and probe signals. The second mechanism is SHB, it is intraband process with very fast lifetime of about (50 -100 fs). CH is last mechanism, this process is intraband process, and it leads to increase the carrier temperature above the temperature of the lattice. After of sub-picosecond, the hot-carrier distributions relax to the lattice temperature by the emission of optical phonons [3].

CH in semiconductor was not taken enough attention, so that carrier dynamic and carrier temperature in QW SOA have been studied theoretically. The effect of doping, pulse effect, and free carrier absorption has been taken in our computation, which did not study earlier as a best of ours our knowledge. . We refer that the software are designed and modeled using a matlab environment,

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In this model, the semi-classical density-matrix equations are modified to take the contribution of carrier heating effects. The equations essentially treat the material as being composed of an inhomogeneous broadened. The carrier distributions approach equilibrium due to carrier scattering process, which is described by phenomenological time relaxation terms. The equation of motion is given as [4].

$$\frac{d\rho_{x,k}(t)}{dt} = -\frac{\rho_{x,k}(t)-f_{x,k}(t)}{\tau_c} - \frac{\rho_{x,k}(t)-f_{x,k}^L(t)}{\tau_{CH}} - \frac{\rho_{x,k}(t)-f_{x,k}^{eq}(t)}{\tau_s} - \frac{i}{\hbar}[d_k^* \rho_{cv,k}(t) - d_k \rho_{vc,k}(t)]E(z,t) + A_{x,k} \quad (1)$$

ρ_{α} denotes the carrier's occupation probabilities for the state k in the conduction and valence bands, respectively, $\rho_{cv,k}$ is proportional to the corresponding atomic polarization. Only vertical interband transitions between conduction and valence band states with the same k-vector are allowed. eq. (1) contains various relaxation and pumping terms. The first term on the right-hand side of eq.(1) describes the relaxation of the carrier distribution function $\rho_{\alpha,k}(\alpha=c,v)$ toward a Fermi distribution function with a time constant τ_c , The second term on the right-hand side of eq.(1) represents the relaxation of the carriers temperature toward the lattice temperature T_L due to scattering process with a characteristic time constant T_{CH} where $f_{x,k}^L$ Fermi function which is defined by the carrier density and the temperature of lattice T_L . The third term describes the relaxation toward the thermal equilibrium distribution $f_{\alpha,k}^{eq}$ due to spontaneous radiative and nonradiative recombination process with a characteristic time constant τ_s . The forth term represents stimulated emission and absorption and finally ($A_{\alpha,k}$) is the pump rate. According to density matrix theory, the carrier density and energy density U_{α} are defined as [4]

$$N_w(t) = \frac{1}{V} \sum_k \rho_{\alpha,k}(t) \quad (2)$$

$$U_\alpha(t) = \frac{1}{V} \sum_k \rho_{\alpha,k}(t) E_{\alpha,k} \quad (3)$$

Using the definition in eqs. (2 and 3), and summing up (1) over k , one obtains [4]

$$\frac{dN}{dt} = \frac{I}{ev} - \frac{N}{\tau_s} - \frac{i}{\hbar v} \sum_k d_k [\rho_{cv,k}(t) - \rho_{vc,k}(t)] E(z,t) \quad (4)$$

$$\frac{dU_\alpha}{dt} = \frac{I \langle E_\alpha \rangle}{eV} - \frac{U_\alpha}{\tau_s} - \frac{U_\alpha - U_\alpha^L}{\tau_{CH}} - \frac{i}{\hbar v} \sum_k E_{\alpha,k} d_k [\rho_{cv,k}(t) - \rho_{vc,k}(t)] E(z,t) \quad (5)$$

$\langle E_\alpha \rangle$ is the averaged energy of the injected carriers, and U_α^L is the temperature of lattice. eqs.(4 and 5) can be simplified, the results are [4].

$$\frac{dN}{dt} = \frac{I}{ev} - \frac{N}{\tau_s} - \nu_g g S \quad (6)$$

$$\frac{dU_\alpha}{dt} = \frac{I \langle E_\alpha \rangle}{eV} - \frac{U_\alpha}{\tau_s} - \frac{U_\alpha - U_\alpha^L}{\tau_{CH}} - g E_{\alpha,0} \nu_g S \quad (7)$$

Where I the injected current, N the effective number of WL states per volume, V the volume of active region, g the total gain ν_g group velocity, S is the photon density which is given by the following equation [4]

$$S(z,t) = \frac{2\epsilon_0 n_g |A(z,t)|^2}{\hbar \omega_0} \quad (8)$$

n represents the reflective index To calculate the rate of carrier temperature, considering the energy density is a function of carrier density and temperature $U_\alpha = U_\alpha(N_\alpha, T_\alpha)$ [4].

$$\frac{dU_\alpha}{dt} = \left(\frac{dU_\alpha}{dN} \right)_T \frac{dN}{dt} + \left(\frac{dU_\alpha}{dT} \right)_N \frac{dT}{dt} = \langle \Delta U_\alpha \rangle \frac{dN}{dt} \quad (9)$$

substituting eqs (6 and 7) in eq (9), the rate of carrier temperature is derived as [4]

$$\frac{dT_\alpha}{dt} = \left(\frac{dU_\alpha}{dT_\alpha} \right)^{-1} \left\{ \left[\Delta E_e - \left(\frac{dU_\alpha}{dN} \right) \right] \frac{I}{ev} + \left[\left(\frac{dU_\alpha}{dN} \right) - \langle U_\alpha \rangle_{stim} \right] \nu_g g (N, T_\alpha) S \right. \\ \left. + \left[\left(\frac{dU_\alpha}{dN} \right) - \langle U_\alpha \rangle_{sp} \right] B N^2 + \hbar \omega \nu_g N \sigma_{fac} S \right. \\ \left. + \left[\left(\frac{dU_\alpha}{dN} \right) + E_g \right] R_{Auger} - \frac{U_\alpha - U_\alpha^L}{\tau_{CH}} \right\} \quad (10)$$

$$\Delta E_\alpha = \frac{m_h}{m_e + m_h} (E_{br} - E_g) \quad (11)$$

$$\langle U_\alpha \rangle_{stim} = \frac{m_h}{m_e + m_h} (\hbar \omega - E_g) \quad (12)$$

$$\langle U_\alpha \rangle_{sp} = \frac{U_\alpha}{N} \quad (13)$$

E_{br} is the energy gap of barrier, E_g is the energy gap quantum well, R_{Auger} is the Auger recombination rate ($R_{Auger} = CN^3$). It should be noted that free carrier absorption term was phonologically added ($\hbar \omega \nu_g N \sigma_{fac} S$), where σ_{fac} is the cross section of free carrier absorption. The solution of eq. (10) can be calculated depending on energy density solution [4],

$$U_\alpha(T_\alpha) = N_w \kappa_\beta T_\alpha \frac{F_1(\eta_\alpha)}{F_0(\eta_\alpha)} \quad (14)$$

Where κ_β is the Boltzmann constant, $F_j(\eta_\alpha)$ is the Fermi-Dirac integral of j-order, The partial derivation of U_α respected with temperature and carrier density is given as [5],

$$\frac{\partial U_\alpha}{\partial T_\alpha} = N \kappa_\beta T_\alpha \left[\frac{F_1(\eta_\alpha)}{F_0(\eta_\alpha)} + \eta_\alpha \left(\frac{F_1(\eta_\alpha) F_{-1}(\eta_\alpha)}{F_0^2(\eta_\alpha)} - 1 \right) \right] \quad (15)$$

$$\frac{\partial U_\alpha}{\partial N} = \kappa_\beta T_\alpha \frac{F_0(\eta_\alpha)}{F_{-1}(\eta_\alpha)} \quad (16)$$

III. ENERGY LEVELS IN QW

As in the bulk material, it has been assumed the energy levels are measured relative to the appropriate band edges. Using the parabolic band model, the energy band in CB is given by the following equation [6]

$$\frac{m_{ch}}{m_{cw}} \sqrt{\frac{\Delta E - E_{ci}}{E_{ci}}} = \begin{cases} \tan & n \text{ even} \\ -\cot & n \text{ odd} \end{cases} \left(\frac{L_z \sqrt{2m_{cw} E_{ci}}}{2\hbar} \right) \quad (17)$$

The energy band model for a single QW is shown in figure(1)

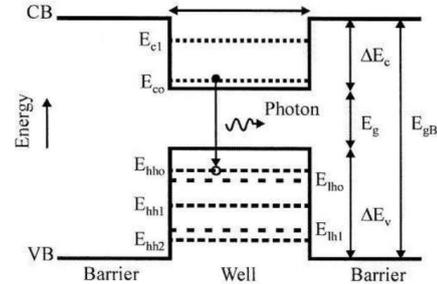


Figure.(1): energy sub-band in QW [6].

where m_{ch} and m_{cw} are the electron effective masses in the well and barrier respectively, eq.(17) generally numerical solution. The maximum of bounded electron states N_c in the well satisfies the conduction [6]

$$\left(\frac{L_z \sqrt{2m_{cw} E_{ci}}}{\pi \hbar} \right) < N_c \quad (18)$$

and using the approximated in the literature [7], the energy levels in CB is given as

$$E_{ci} = \frac{\left[\frac{a_c (1+i)\pi}{2(L_z + \Delta L_c)} \right]^2}{\left\{ 1 + \left[\frac{(1+i)\pi}{2} \right]^2 b_c \left(\frac{\Delta L_c}{L_z + \Delta L_c} \right)^3 \right\}} \quad (19)$$

where $\Delta L_c = \frac{a_c}{\sqrt{b_c \Delta E_c}}$, $a_c = \frac{2\hbar}{\sqrt{2m_{cw}}}$ and $b_c = \frac{m_{cw}}{m_{ch}}$

by similarity, it can be used the set of equations to compute the energy levels of light and heavy hole A similar set of equation (Ehhj and Elhj) by replacing ΔE_c with ΔE_v and m_{cw} and m_{cb} by m_{lhw} and m_{hbw} . under bias condition the quasi-fermi levels E_{fc} and E_{fv} in the CB and VB respectively, the electron density n and hole density p in the well by [6]

$$n = \frac{m_{cw} KT}{n \hbar^2 L_z} \left[\sum_{i=0}^{N_c-1} \ln(1 + \exp((E_{fc} - E_{ci})/KT)) \right] \quad (20)$$

$$p = \frac{KT}{n \hbar^2 L_z} \left[\sum_{i=0}^{N_{hh}-1} m_{hhw} \ln(1 + \exp((E_{fv} - E_{hi})/KT)) + \sum_{i=0}^{N_{lh}-1} m_{lhw} \ln(1 + \exp((E_{fv} - E_{li})/KT)) \right] \quad (21)$$

Where Nhh (Nlh) are the number of bounded heavy-hole (light-hole).

IV. RESULT AND DISCUSSIONS

In a semiconductor material, the CH can be studied by the nonlinear gain coefficients and carrier temperature. In this investigation, the carrier temperature of QW SOA has been studied numerically, depending on density matrix theory and the carrier dynamics in QW. The doping effect, pulse shape, free absorption coefficient and CH relaxation . The parameters used in this simulation are listed in the following table.

TAB.(1): PARAMETERS FOR NUMERICAL COMPUTATION USED IN THE SIMULATION.

| symbol | Name | value/ unit | references |
|----------------|----------------------------------|-----------------------------|------------|
| τ_{WR} | nonradiative relaxation | 0.4 ps | 4 |
| LW | Thickness of active layer | 8 nm | 8 |
| τ_{CH} | carrier heating relaxation | 2.5 ps | 8 |
| τ_{SHB} | spectral-hole burning relaxation | 150 fs | 9 |
| me | effective electron mass | 0.041 m0 | 4 |
| mh | effective hole mass | 0.38 m0 | 4 |
| TL | lattice temperature | 300 K | 10 |
| ΔE | energy difference | 80 meV | 11 |
| σ_{fac} | free carrier absorption | $3.5 \times 10^{22} m^{-2}$ | 4 |

V. DYNAMIC OF CARRIER IN QW SOAs

The dynamics of carrier in QW SOAs have been studied depending on the rate equations of electron and the theory of pulse propagation in QW. The modeling involves studying of electron occupation probability, time recovery, current injection, nonradiative rate and pulse shape effect.

The dynamic of carrier in QW is simulated in figures (2-6), the time recovery increasing straightforward with nonradiative recombination, and injected current (see figure (2) and figure (3) respectively). After 2 ps of time response; the steady-state solution of occupation probability reaches the maximum at 750 mA as shown in figure (3). Also, the trends in figure (2) show the nonradiative recombination relaxation time reduces the rate of carrier occupation in quantum states. The pump pulse shape effect, represented by the full width of half maximum, on time recovery of QW SOAs devices is illustrated in figure (4), the

electron occupation probability decreases with width of injected pulses. Figures (5 & 6) show behave of time recovery versus pulse width and nonradiative rate, the trends of time recovery are decreasing exponentially with pulse width and increasing exponentially with nonradiative rate.

The time series solution and time recovery in QW SOAs are about sub of nanosecond it is a long time than these values in the QD (subpicosecond) [12-13]. The fast relaxation of intraband in QD gives the system fast relaxation, and then it will enhance the performance of QD devices. The results showed that; the occupation probability in QW is in agreement with the behavior of dynamic in QD state [14, 15].

The effect of the nonlinear gain coefficient due to SHB on the occupation of WL is studied, with ($\epsilon_{SHB} = 0.5 \times 10^{-23} m^{-3}$) [4]. The nonlinear gain coefficient reduces the photon density by the factor $(1 + S \times \epsilon_{SHB})^{-1}$, and consequently effected on the rate of carrier density and time recovery.

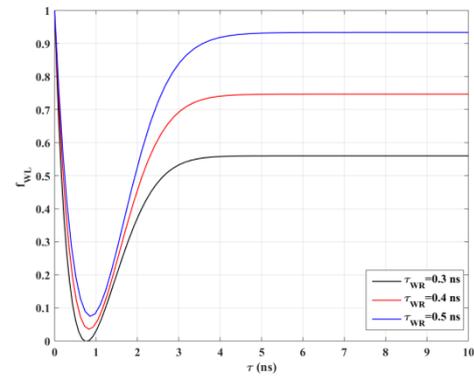


Figure:(2): occupation probability versus nonradiative relaxation in QW.

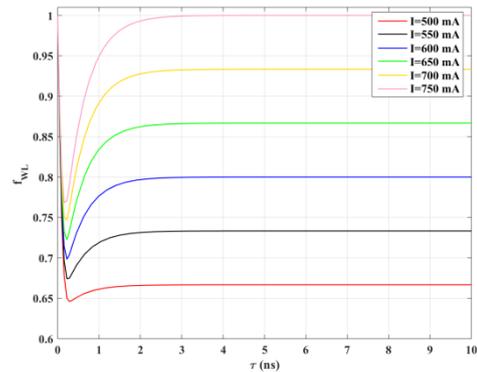


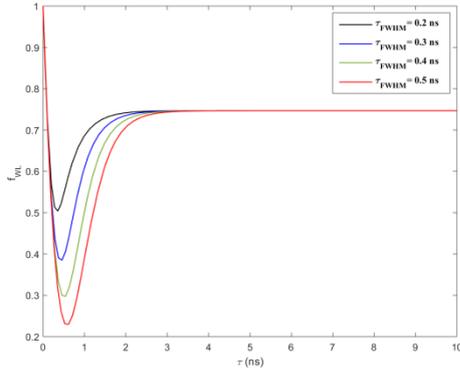
Figure:(3): the effect of carrier current injection on time series of occupation probability.

VI. CARRIER TEMPERATURE IN QW

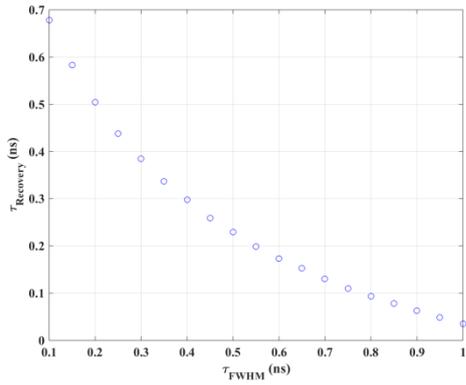
The carrier temperature in QW has been formalized, the formula (Eq.(10)) involves all effective QW parameters such as; injected pump, carrier heating relaxation, spontaneous carrier relaxation, and free carrier absorption. In this investigation, the carrier temperature has been studied through the effects of nonlinear gain coefficients, carrier heating relaxation and lattice temperature.

The effect of Ne--type ionized donor per QW layer on the carrier temperature is studied in figure (8), with increases carrier concentration, the collision between the carrier will be increase, and then it will cause the raise carrier temperature as shown in figure (9). Free carrier absorption is considered a very effective source in CH effects, the carrier temperature is straightforward with σ_{fca} as shown in figure (10). The contribution of carrier heating that is represented by carrier heating relaxation is studied in figure (11). As expected the behave in QD SOA, where the carrier density and occupation are increase with a long life time of CH relaxation, and then the collision of carrier in wetting layer will raise the carrier temperature [12]. We refer that; the results in figures (8-12) are in agreement with literatures [4, 10, and 16].

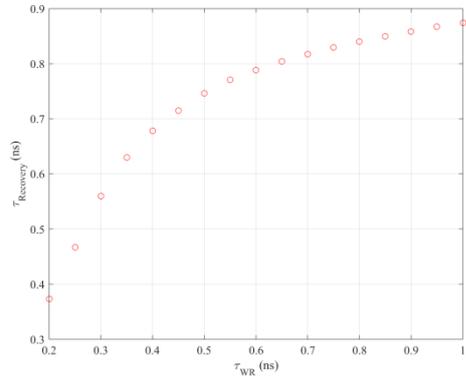
It is well known, the short pump pulse passes through semiconductor materials, the system will be in excited state, and the carrier is rapidly removed from QW states. Then the system will become at non-equilibrium, consequently, the relaxation process of carriers from the reservoir to the QW states began as a result of the pulse effect. In this investigation, the pulse effect represented by FWHM on performance of SOA is studied (see Figure. (12)), the temperature of carriers is directly increased with the full width of half maximum of injected pulses. Figure (13) shows the thermal equilibrium between lattice temperature and reservoir carrier temperature, the carrier temperature at low temperature takes the same time to relax to lattice temperature (TL), these results is a contradiction in QDs [11].



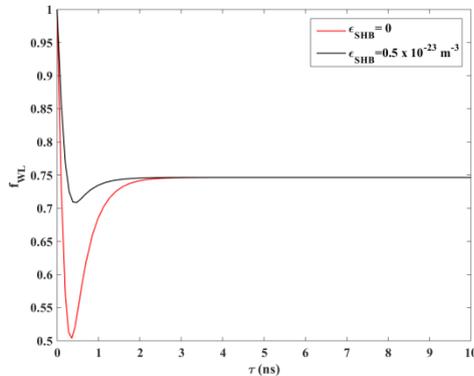
Figure(4): the of occupation probability versus pulse shape.



Figure(5): time recovery of QW versus pulse width at half maximum.



Figure(6): time recovery of QW nonradiative relaxation.



Figure(7): effect of ϵ_{SHB} on time series of occupation probability.

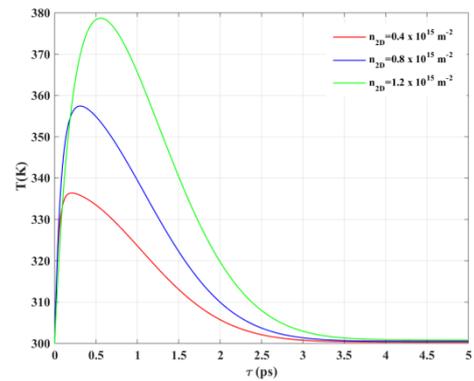


Figure (8): time series of carrier temperature for many values of surface carrier concentration.

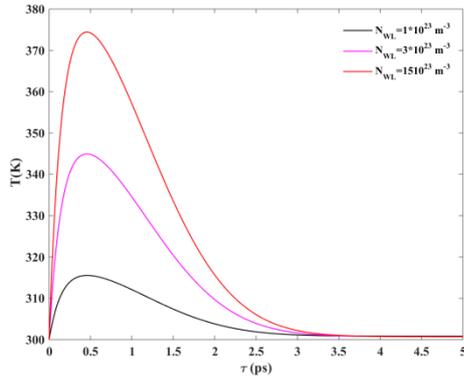


Figure (9): the time series of carrier temperature for different values of carrier density

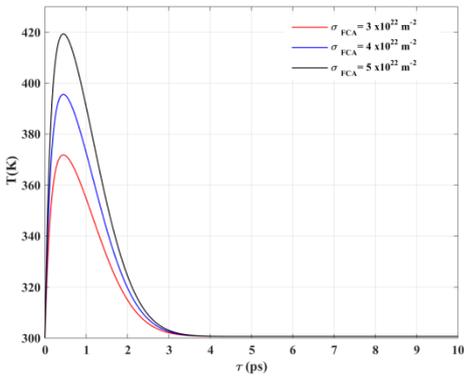


Figure (10): effect of free carrier absorption on carrier temperature response

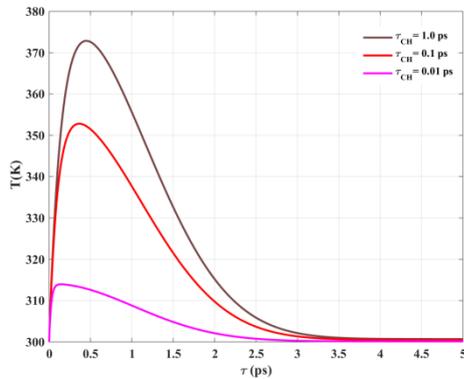


Figure (11): effect of carrier heating relaxation on carrier temperature response

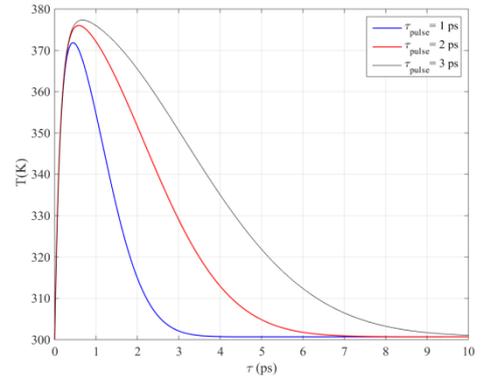


Figure (12): carrier temperature response: pulse effect

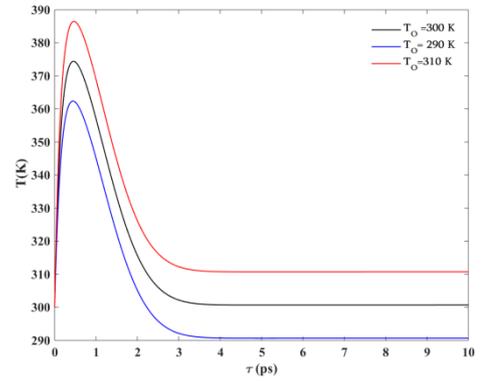


Figure (13): time series of carrier temperature for many values of lattice temperature.

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