ISSN 1991- 8690 الترقيم الدولي ٨٦٩٠ - ١٩٩١
website : http:// jsci.utq.edu.iq Email: utjsci@utq.edu.iq

Modes Analysis in Step Index Fibers

Sukina T. Gafel [(1)] Hashim Ali Yusr [(1)] Hassan Abid Yasser[(2)]

[(1)]Wasit University- Science College [(2)] ThiQar University- Science College

Abstract

The modes are solutions of Maxwell's equations depending on boundary conditions of the waveguide (step-index fiber with $n = n_1$ for core and $n = n_2$ for cladding). In this paper, the general characteristic equation was deduced. This equation was used to determine modes TE, TM, and the hybrid modes EH and HE. Also, the linear polarization modes of the weakly guiding approximation $n_1 \approx n_2$ were illustrated. The results proved that: the mode spot size, the propagation constants, the eigenvalues, and the nonlinearity depend on the mode order and the normalized frequency. Thereafter, the simulation proved that the number of modes is related to the normalized frequency in a quadratic function.

Keywords: Maxwell's equations, fiber modes, normalized frequency.

تحليل الأنماط في الألياف البصرية ذات معامل الانكسار المتغير فجائيا

الخلاصة

الأنماط هي حلول لمعادلات ماكسويل باعتماد الشروط الحدودية لدليل الموجة (ليف بصري ذو معامل انكسار n_1 للقلب و n_2 للمحيط). تم في هذا البحث استنتاج المعادلة المميزة العامة ومنها تم تحديد الأنماط المستعرضة كهربائيا TE والمستعرضة مغناطيسيا TM والهجينة EH و HE، وكذلك تم تحديد الحلول للحالة التقريبية $n_1 \approx n_2$ والتي تعطي ما يسمى بالاستقطاب الخطي LP. أثبتت النتائج أن: تركيب بقعة النمط، ثابت الانتشار، القيم الذاتية، واللاخطية تعتمد على مرتبة النمط وأن عدد الأنماط يرتبط بعلاقة تربيعية مع تردد القطع المعاير والتي تم حسابها خلال المحاكاة.

1.Introduction

The fact is that light can propagate inside an optical fiber only as a set of separate beams, or rays. In other words, if we were able to look inside an optical fiber, we would see a set of beams traveling at distinct propagating angles, α, ranging from zero to the critical value, α_c [1]. These different beams are called modes. We distinguish modes by their propagating angles and we use the word order to designate the specific mode. The rule is this: the smaller the mode's propagating angle, the lower the order of the mode. Thus, the mode traveling precisely along the fiber's central axis is the zero-order mode and the mode traveling at the critical propagation angle is the highest order mode possible for this fiber [2]. The zero-order mode is also called the fundamental mode. Many modes can exist within a fiber, and so a fiber having many modes is called a multimode fiber [3]. Recently, silica fibers have been produced that permit the transmission of optical signals over several kilometers. In general, these fibers support many modes, which propagate at different velocities. Since this causes signal distortion over long distances, fibers that transmit only a limited number of modes are of special interest [4]. A fiber waveguide consists of a thin central glass core surrounded by a glass cladding of slightly lower refractive index. Most modes can be suppressed by making the core thin and the index different between core and cladding small. Typically, a difference of a few parts in a thousand is feasible [5]. This avoids propagation of most modes. The modes that do propagate are weakly guided, but in general the guidance is sufficient to negotiate bends with radii of tens of centimeters [6]. Maxwell's equations have exact solutions for the dielectric cylinder, but even with the simplifying assumption that the cladding be infinitely thick these solutions are too complicated to be evaluated without computer. Recent efforts in simplifying the theory for weakly guided modes had promising results, but in the region of interest, they did not lead to the kind of simple formulas one would wish to have for fiber design work [7,8]. The following paper is aimed at such formulas and functions. It is meant as a help for engineering applications directed toward fiber communication systems. Most results are valid for all frequencies and propagation conditions-even at cutoff-with an accuracy of the order of the index difference between core and cladding [9].In this paper, the general characteristics equation of modes in step-index fiber was analyzed. It was reduced to simple forms for the cases TM and TM modes. Also, the weakly guiding property give a generalized characteristics equation to conclude the propagated modes in step-index fibers. The properties of the modes were examined under the constraints of the characteristics equation.

2.The Characteristic Equation

Now, the wave propagation equation will be illustrated but the vectorial nature of light will be not assumed. Instead of solving Maxwell's equations, the solution of the following Helmholtz equation in cylindrical coordinates will be discussed [9]

$$\frac{\partial^2 F}{\partial r^2}+\frac{1}{r}\frac{\partial F}{\partial r}+\frac{1}{r^2}\frac{\partial^2 F}{\partial \varphi^2}+\frac{\partial^2 F}{\partial z^2}+n^2k_0^2F=0 \qquad (1)$$

where r, φ, and z are respectively the polar, the azimuthal and the axial coordinates; F is the field; $k_0=2\pi/\lambda$ is the wavenumber in vacuum; the index of refraction for step-index fiber n is n_1 for $r<a$ and n_2 for $r>a$ where a is the core radius. The electric and magnetic fields must be continuous and so must be their derivatives [10]. Translating this concept to the scalar theory of light, the components F and $\partial F/\partial r$ must be continuous. Depending on the continuity and the boundary conditions, the solutions of Eq.(1) are [8]

$$F(r,\varphi,z)=e^{ikz}e^{i\ell\varphi}J_\ell(kr) \quad , \quad r<a \qquad (2a)$$
$$F(r,\varphi,z)=e^{ikz}e^{i\ell\varphi}K_\ell(\gamma r) \quad , \quad r>a \qquad (2b)$$

where ℓ is an integer number, J_ℓ is the Bessel function of the first kind, K_ℓ is the modified Bessel function of second kind, and [11]

$$k^2=n_1^2k_o^2-\beta^2 \qquad (3a)$$
$$\gamma^2=\beta^2-n_1^2k_o^2 \qquad (3b)$$

where k is the longitudinal propagation constant, γ is the transverse propagation constant, and β is the eigenvalue. The Bessel functions are chosen so that F is always limited and approaches zero as $r\to\infty$. The choice of k and γ is not free, indeed the boundary conditions at $r=\infty$ implies that [12]

$$\left[\frac{J_\ell'(X)}{XJ_\ell(X)}+\frac{K_\ell'(Y)}{YK_\ell(Y)}\right]\left[n_1^2\frac{J_\ell'(X)}{XJ_\ell(X)}+n_2^2\frac{K_\ell'(Y)}{YK_\ell(Y)}\right]=\left[\frac{\ell\beta V^2}{k_oX^2Y^2}\right]^2 \qquad (4)$$

where $X=ka$ and $Y=\gamma a$. Eq.(4) is called the characteristics equation. An important quantity for fibers that is the normalized frequency parameter can be defined as [11]

$$V=\sqrt{X^2+Y^2}=a\sqrt{k^2+\gamma^2}=ak_o\sqrt{n_1^2-n_2^2}=k_oaNA \qquad (5)$$

If one wants to has solutions with k and γ real, the intersection of the right hand side and the left hand side of Eq.(4) with $X<V$ must be found. For each ℓ we have different intersection points that are M_ℓ in number. When ℓ increases M_ℓ decreases until it become zero, we call ℓ_{max} the value of ℓ for which Eq.(4) has no solution ($M_\ell=0$). It is interesting to notice that for $\ell=0$ we always find at least one solution for any value of V, when $V<2.405$ only one mode is present and the fiber is a monomodal waveguide. This would not be possible in a waveguide made with reflective walls whose boundary conditions impose that the field must be zero at $r=a$. We remark that Eq.(4) has also imaginary solution of X that will give raise to imaginary values of k that corresponds to evanescent waves.

3. Fiber Modes

A graph of Eq.(4) in $(\gamma a, Ka)$ coordinates is a combination of Bessel function curves. The graph of Eq.(5) is a circle with radius V. The intersects between the two curves are solutions. Each intersect is designated as a mode that can be excited and gives such information as the cutoff conditions and the cross sectional distribution of light in the fiber. The solutions of the characteristic equation vary a great deal depending on whether $\ell = 0$ or $\ell \neq 0$.

For meridional rays that pass through the fiber axis $\ell = 0$, the characteristics equations of TE and TM modes, i.e. Eq.(4), become

$$\left[\frac{J_1(X)}{XJ_0(X)} + \frac{K_1(Y)}{YK_0(Y)}\right] = 0 \quad (6a)$$

$$\frac{n_1^2}{n_2^2}\left[\frac{J_1(X)}{XJ_0(X)} + \frac{K_1(Y)}{YK_0(Y)}\right] = 0 \quad (6b)$$

The difference between these equation is the ratio $(n_1/n_2)^2$. In the weak guiding approximation $n_1 \approx n_2$, these two modes will be degenerate. Note that, in the TE modes we have the electric field $E_z = 0$, while in the TM modes we have the magnetic field $H_z = 0$.

In general, for $\ell \neq 0$ the modes is called as skew rays, and formal notation is HE and EH. For weakly guiding approximation, the linear polarization (LP) modes can be expressed as sum of TE, TM, HE, EH that become degenerate for small $\Delta n = n_1 - n_2$. In this case, Eq.(4) will be

$$\frac{XJ_{m-1}(X)}{J_m(X)} + \frac{YK_{m-1}(Y)}{K_m(Y)} = 0 \quad (7)$$

where m is defined as: 1 for TE and TM modes, $\ell + 1$ for EH modes, and $\ell - 1$ for HE modes. However, the LP modes are related to the other modes representation as [13]

$$\begin{aligned} LP_{01} &= HE_{1p} \\ LP_{1p} &= HE_{2p} + TE_{0p} + TM_{0p} \quad (8) \\ LP_{mp} &= HE_{m+1p} + EH_{m-1p} \ , \quad m \geq 2 \end{aligned}$$

The cutoff frequency for each propagation mode can be calculated from Eq.(7). A mode is said to be cut off when its field ceases to be evanescent in the cladding. Hence, the phase velocity of the field becomes equal to that of a plane wave propagating in the cladding material. Accordingly, the cutoff frequency is found by setting γ equal to zero. In general, the cutoff frequency are obtained via letting $Y \to 0$ and therefore $X \to V$. For TE and TM modes they are equal to the positive roots of $J_0(V) = 0$. So if r_{0p} is the pth positive root of $J_0(V)$ for $p \geq 1$, the cutoff frequency for TE_{op} and TM_{op} is $V_0 = r_{op}$. For example TE_{01} and TM_{01} have the cutoff frequency $r_{01} = 2.405$. For $EH_{\ell p}$ modes the cutoff frequencies are obtained via the equation $J_\ell(V) = 0$ for $\ell \geq 1$ and $p \geq 1$. So the cutoff frequency of the mode $EH_{\ell p}$ is $V_0 = r_{\ell p}$ which is the pth positive root of $J_\ell(V)$.

The normalized propagation parameter is given by $b = (Y/V)^2$ [10]. An advantage of taking b as a variable is that b has a finite range from 0 to 1. Using this substitution, the value of $ka = X$ is calculated from b as $X = \sqrt{1-b}V$ and the characteristics equation will be

$$\frac{V\sqrt{1-b}J_{m-1}(V\sqrt{1-b})}{J_m(V\sqrt{1-b})} + \frac{V\sqrt{b}K_{m-1}(V\sqrt{b})}{K_m(V\sqrt{b})} = 0 \quad (9)$$

The last equation may be used to determine the solutions in terms the normalized propagation constant and the normalized frequency.

The effective mode area is defined as [7]

$$A_{eff} = \frac{[\int_0^{2\pi}\int_0^{\infty} |F|^2 \, r dr d\varphi]^2}{\int_0^{2\pi}\int_0^{\infty} |F|^4 \, r dr d\varphi} \quad (10)$$

where F is the field distribution that determines from Eq.(2) by setting $z = 0$. The parameter A_{eff} is very important to compute the nonlinearty factor $\Gamma = w_0 N_2 / cA_{eff}$, where N_2 is the third-order nonlinear refractive index. Since each mode has different distribution, such that the nonlinearity factor is a function of mode index.

4. Results and Discussion

Our simulation is characterized by many numbers of different parameters, for completeness, we list a typical parameter set below

$n_1 = 1.5$, $n_2 = 1.49$, $\lambda_0 = 1550$, $N_2 = 2.6 \times 10^{-20}\,\mathrm{m}^2/\mathrm{W}$

There are another parameters that will be changed through simulation depending on the mode order and the normalized frequency. Figs.(1) and (2) represent the logitudenal and the transverse power distribution of the mode spot for different mode orders that are calculated using the relations Eqs.(2) and (9). It is clear that the mode distribution becomes more complicated if the mode order increases. This complication is happen by increasing the Bessel function order or by increasing the root order of a certain Bessel function. The spot size, in general, is more expanded for the higher order modes, so a part from mode energy will be outside the fiber core. As a consequence, the fundamentl mode LP_{01} has the larger power that will be confined in the core. In other words, the higher order modes is the higher energy loss.Fig.(3) obtains the number of roots and the number of modes as function of the normalized frequency. The number of roots changes as linear function with the normalized frequency. That is; the increasing of normalized frequency will increase linearly the number of roots, but there are many degenerate modes that concide with each other, this will make the number of modes as a quadritic function of normalized frequency. This quadritic form is calculated to be: $M = 0.5V^2 + 0.38V + 0.22$. However, the appoximated analytic form in the scientific litriture is $M = 0.5V^2$, see [5,11].

Fig.(4) explains the parameters: k, γ, β, and A_{eff} as functions of normalized frequency. Since $V^2 = a^2(k^2 + \gamma^2)$, then the increasing of k will decrease γ for the same V and vice versa and this cirtianly depends on the mode order. The propagation constant k decreases with V, this behavior is attributed to the fact that the core radius is increased and as a result the mode may be apart from the fiber axis. This reduction must be associated with increasing the γ value. Also, increasing the mode order will raise the k value and reduce the γ value. This behavior is attributed to the distinction of the mode spot size with respect to the core dimension. The eigenvalue inceases slightely with the normalized frequency for the lower modes, but the change will be strong at the higher order modes. This is an acceptable fact where the the lower order modes are propagted near the fiber axis and vice versa. The effective area of the modes increases with normalized frequency. This behavior is attributed to the increasing in core radius that increases the confinement factor of mode. The higher order modes have the same behavior but the satisfied effective areas are small. In general, the effective area is decreased with increasing normalized frequency or mode order. So, the nonlinearity factor is higher for higher order modes and for smaller normalized frequency.

5. Conclusions

We conclude this paper by: the higher order modes have a more complicated spot size and vice versa. The increasing of normalized frequency will increase the number of modes. The propagation constants and eigenvalues are slightly affected by normalized frequncy variation for the lower order modes, while the affection is strog for the higher order modes. This means that the mode tend to propagte apart from fiber axis by increasing its order. Each mode faces a nonlinearity that inceases with its order.

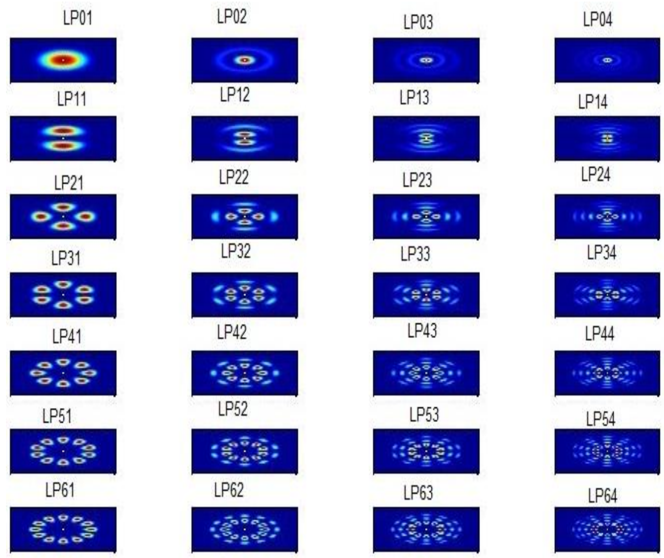

Fig.(1): the trasverse power distribution for different modes.

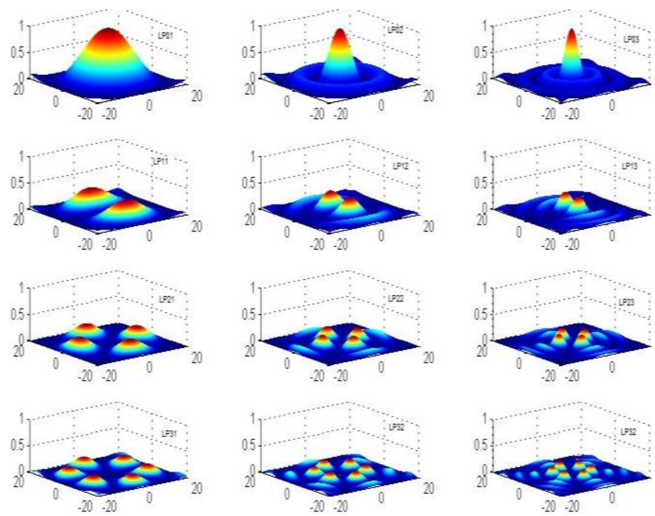

Fig.(2): the longitudinal distribution of power for different modes.

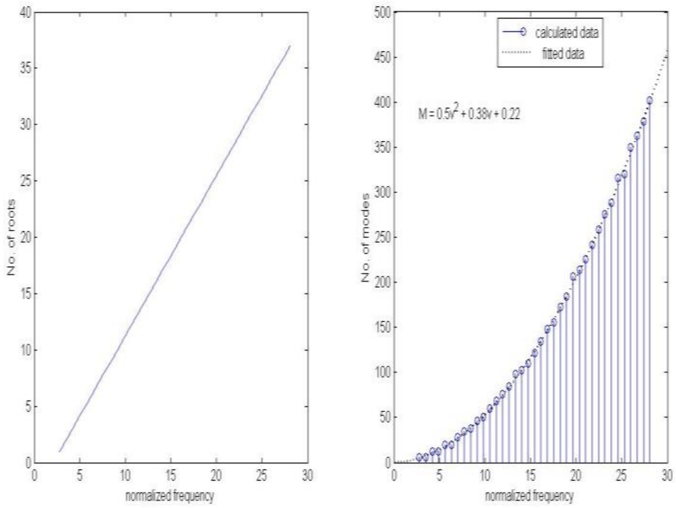

Fig.(3): the number of roots and number of modes as functions of normalized frequency.

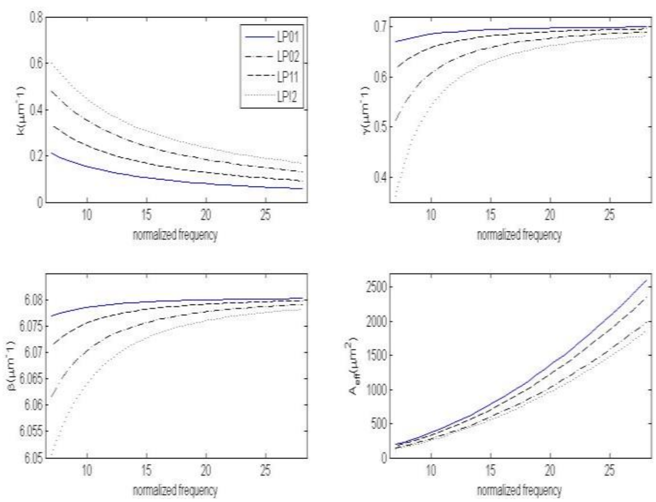

Fig.(4): β , k , γ , and A_{eff} as functions of normalized frequency.

References

[1] C. DeCusatis and C. J. Sher DeCusatis, Fiber Optic Essentials, Elsevier, 2005.

[2] J. Hecht, Understanding Fiber Optics, Prentice Hall, 5th ed. 2005.

[3] R. P. Khare, Fiber Optics and Optoelectronics, Oxford University Press, 2004.

[4] B. K. Garside, T. K. Lim, T. K. , and J. P. Marton, “Propagation characteristics of parabolic-index fiber modes: Linearly polarized approximation,” J. Opt. Soc Amer., vol. 70, no. 4, 1980.

[5] E. A. J. Marcatili D. Gloge. Multimode theory of graded-core fibers. The Bell System Technical J, 1973.

[6] D. Marcuse, Loss analysis of single mode fiber splices, Bell System Technical J, vol.56, no.5, 1977.

[7] J. A. Buck, Fundamentals of Optical Fibers, Wiley, 1995, 2nd ed. 2004.

[8] C.-L. Chen, Elements of Optoelectronics and Fiber Optics, Irwin, 1995.

[9] M. C. Teich B. E. A. Saleh. Fundamentals of Photonics. Wiley, 2007.

[10] D. Marcuse, Theory of Dielectric Optical Waveguides, Academic Press, San Diego, CA, 1991.

[11] J.A. Buck, Fundamentals of Optical Fibers, Wiley, New York, 1995.

[12] S. Bottacchi, Multi-Gigabit Transmission Over Multimode Optical Fiber. New York: Wiley, 2006.

[13] S. Fan and J. M. Kahn, “Principal modes in multimode waveguides,” Opt. Let., vol. 30, no. 2, 2005.