ISSN 1991- 8690 الترقيم الدولي ٨٦٩٠ - ١٩٩١

website :http:// jsci.utq.edu.iq Email: utjsci@utq.edu.iq

Differential Quadrature Method for solving the linear and nonlinear Klein - Gordon Equation

Ahmed J. Hussein

Math.Dept., education Coll., University of Thi-Qar

Abstract

In this paper, numerical solution of Klein –Gordon equation is obtained by using differential quadrature method (DQM).This method is demonstrated by several examples. we compare the results with exact solution our results show this method is powerful and efficient for solve Klein –Gordon equation.

Keywords: Differential Quadrature method, Klein –Gordon equation finite difference method.

طريقة التفاضل التربيعي لحل معادلة كلين –جوردن الخطية واللاخطية

احمد جبار حسين

كلية التربية للعلوم الصرفة- جامعة ذي قار

الخلاصة

في هذا البحث تم حل معادلة كلين–جوردن باستخدام طريقة التفاضل التربيعي هذه الطريقه زودت بعدد من الامثلة ، قمنا بمقارنة النتائج التي حصلنا عليها باستخدام هذه الطريقة مع الحل الحقيقي لهذه المعادلة واظهرت النتائج كفاءة الطريقة واهميتها لحل معادلة كلين–جوردن .

1. Introduction

It is well known that many phenomena in scientific fields such as solid state physics, plasma physics, fluid dynamics, mathematical biology and chemical kinetics, can be modeled by nonlinear partial differential equations. The nonlinear models of real-life problems are still difficult to solve either numerically or theoretically. Abroad class of analytical solutions methods and numerical solutions methods were used to handle these problems (Wang, 1988; Jeffery and Mohamed, 1991; Wadati, 1972).

Consider the nonlinear Klein –Gordon equation of the form:

$$\frac{\partial^2 u(x,t)}{\partial t^2}+u_{xx}(x,t)+bu(x,t)+G\big(u(x,t)\big)=f(x,t) \qquad (1)$$

$$u(x,0)=a_0(x), \quad u_t(x,0)=a_1(x) \qquad (2)$$

Where b is a real number, G is a given nonlinear function, and f is a known function.

The Klein –Gordon equation is one of the more important mathematical models in quantum mechanics(Whitham, 1974;Zauderer, 1983).The equation has attracted much attention in studying solutions in a collision less plasma, the recurrence of initial states, and in examining the nonlinear wave equations (Dodd et al., 1982) . With reference to the numerical solution for this problem we can see many published papers. Many authors (Deeba and Khuri, 1996;El-sayed, 2003;Kaya and El-Sayed, 2004;Wazwaz, 2006) used Adomain's decomposition method for solving linear and nonlinear Klein –Gordon equations. Inc (2006) investigate the special exact solutions of the modified nonlinearly Klein –Gordon-type equations by using some ansatze, and obtained new soliton solution with compact support and solitary pattern solutions having infinite slopes or cusps, solitary wave and periodic solutions. Wazwaz (2006) studied the nonlinear Klein – Gordon equations with power law nonlinearities, used the tenth method for analytic treatment for these equations. The analysis leaded to travelling wave solutions with compactons, solitary patterns and periodic structures.

Another powerful analytic method, called (DQM), was originally developed by simple analogy with integral quadrature, which is derived using the interpolation function (Bellman, R.and Casti, J.1971) and (Bert,C.W and Mailk, M.1996) .Currently; many engineers apply numerical techniques to solve sets of linear algebraic equations. They include the Rayleigh-Ritz method, the Galerkin method, the finite element method, the boundary element method and others, which are applied to solve partial differential equations. Bellmanet al.(Bellman, R. and Cast, J.1971) and (Bellman, R.E, kash B.G. and Casti, J.1997) first introduced the differential quadrature method. His work has been applied in diverse areas of computational mechanics and many researchers have claimed that the differential quadrature approach is a highly accurate scheme that requires minimal computational efforts. Differential quadrature has been shown to be a powerful tool for solving initial and boundary value problems, Thus has became an alternative to existing methods. Bert et al. (Feng, Y. and Bert, C.W. 1992), and (Chen, W.L., Striz, A.G and Bert, C.W., 2000) who investigated the static and free vibration of beams and rectangular plates using the (DQM), proposed the technique that can be applied to the double boundary conditions of plate and beam problems. In this approach boundary points are chosen at a small to ensure the accuracy of the solution, in this paper we introduce (DQM) as a powerful important method and obtained results by this method are excellent and high accuracy where compare with exact solution .by using several examples.

2. Differential Quadrature Method (DQM)

For a function, f(x, t), the (DQM) approximation for the m order derivative at the x_i sample point is given as follows (Han, J.b.and Liew, K.M.1999)

$$\frac{\partial^m}{\partial x^m}\begin{Bmatrix} f(x_1,t) \\ f(x_2,t) \\ \vdots \\ f(x_N,t) \end{Bmatrix} \cong \left[a_{ij}^{(m)}\right]\begin{Bmatrix} f(x_1,t) \\ f(x_2,t) \\ \vdots \\ f(x_N,t) \end{Bmatrix} \; for\; i,j = 1,2,\dots,N. \qquad (3)$$

Where $f(x_i,t)$ the functional value at grid is point x_i, and $a_{ij}^{(m)}$ is the weighting coefficient of the m order differentiation of these functional values. The most convenient technique is to choose equally spaced grid points, but the numerical results were inaccurate in this work when equally spaced points were selected. This finding demonstrates that the choice of a grid points distribution and the test functions markedly influence the efficiency and accuracy of the results in some cases. The selection of grid points always importantly affects solution accuracy. In the numerical experiments, we can used the two types of grid point

Either chebyshev-Gauss-Lobatto grid points (Liew, K.M., Han, J.B. and Xiao, Z.M.1996)

$$x_i = \frac{1}{2}\left(1 - cos\frac{(i-1)\pi}{N-1}\right) \; for\; i = 1,2,\dots,N \qquad (4)$$

Or uniform grid points $x_i = x_1 + ih$, i=1,2,…,N where $x_1 = a$ and $h = (b-a)/h$.

The differential quadrature weighted coefficients can be derived using numerous techniques. To overcome the numerical poor condition in determining the weighted coefficient, $D_{ij}^{(m)}$the following Lagrange interpolation polynomial is introduce (Du, H., Liew, K.M. and Lim, M.K., 1996) .

$$f(x,t) \cong \sum_{i=1}^{N}\frac{M(x)}{(x-x_i)M_1(x_j)}f(x_i,t). \qquad (5)$$

Where

$$M(x) = \prod_{j=1}^{N}(x-x_j)\,,$$

and

$$M_1(x) = \prod_{j=1, i\neq j}^{N}(x-x_j) \; for\; i = 1,2,\dots,N\,.$$

Substituting eq. (5) into eq. (3) yield the following equation s

$$a_{ij}^{(1)} = \frac{M_1(x_i)}{(x_i-x_j)M_1(x_j)} \; for\; i,j = 1,2,\dots,N \; and\; i \neq j, \qquad (6)$$

and

$$a_{ii}^{(1)} = -\sum_{j=1, j\neq i}^{N} a_{ij}^{(1)} \; for\; i = 1,2,\dots,N\,. \qquad (7)$$

Once the grid points are selected, the coefficients of the weighted matrix can be acquired using equations (4) and (5). Notably, the numbers of test functions exceed the highest order of the derivative in the governing equations. High-order derivative of weighted coefficients can also be acquired using matrix multiplication , as follows:

$$a_{ij}^{(2)} = \sum_{k=1}^{N} a_{ik}^{(1)} a_{kj}^{(1)} \quad for\ i,j = 1,2,\dots,N \qquad (8)$$

$$a_{ij}^{(3)} = \sum_{k=1}^{N} a_{ik}^{(1)} a_{kj}^{(2)} \quad for\ i,j = 1,2,\dots,N \qquad (9)$$

$$a_{ij}^{(4)} = \sum_{k=1}^{N} a_{ik}^{(1)} a_{kj}^{(3)} \quad for\ i,j = 1,2,\dots,N \qquad (10)$$

3. Numerical scheme for Klein- Gordon equation

The space derivative in equation (1) is approximated by the polynomial differential quadrature method. This equation can be changed in to the following form:-

$$\frac{d^2u(x_i,t)}{dt^2} = -\left[a_{i1}^{(2)} \quad a_{i2}^{(2)} \ \dots\dots a_{in}^{(2)}\right] u(x_j,t) - bu(x_i,t) - gu(x_i,t) - f(x_i,t) \qquad (11)$$

And equation (2) can be written as:-

$$u(x_j,t) = a_0(x) \quad , \frac{du(x_j,t)}{dt} = a_1(x) \qquad (12)$$

$$for\ i,j = 1,2,\dots,N$$

This system of ordinary differential equation can be solved by using finite difference method or slandered Rung-Kutta method (Rk4) for higher system.

4. Finite difference method

In this section we replacing the time derivative by a central difference scheme

$$u_{tt} = \frac{(u_{i,j+1} - 2u_{i,j} + u_{i,j-1})}{\Delta t^2} \qquad (13)$$

The equation (11) becomes

$$\frac{(u_{i,j+1} - 2u_{i,j} + u_{i,j-1})}{\Delta t^2} = -\left[a_{i1}^{(2)} \quad a_{i2}^{(2)} \dots a_{in}^{(2)}\right] u(x_j,t) - bu(x_i,t) - gu(x_i,t) - f(x_i,t) \qquad (14)$$

$$\to u_{i,j+1} = -\Delta t^2 \left[\left[a_{i1}^{(2)} \quad a_{i2}^{(2)} \dots a_{in}^{(2)}\right] u(x_j,t) - bu(x_i,t) - gu(x_i,t) - f(x_i,t)\right] + 2u_{i,j} - u_{i,j-1} \qquad (15)$$

For j=0 and from the initial condition $u_t(x_i,t) = a_1(x)$ and the central difference scheme $u_k = \frac{u_{i,j+1} - u_{i,j-1}}{2\Delta t}$ we have $\frac{u_{i,1} - u_{i,-1}}{2\Delta t} = a_1(x)$

$$\to u_{i,-1} = u_{i,1} - 2\Delta t\, a_1(x) \qquad (16)$$

Substitution equation (16) into equation (15) we get

$$u_{i,1} = -\frac{(\Delta t^2)}{2}\left[\left[a_{i1}^{(2)} \quad a_{i2}^{(2)} \dots a_{in}^{(2)}\right] u(x_j,t) - bu(x_i,t) - gu(x_i,t) - f(x_i,t)\right] + u_{i,0} + \Delta\, t a_1(x) \qquad (17)$$

Above equation use at j=0 and when j=1, 2, ..., n+1, we use equation (15)

5. Applications

Example 1:

Consider the linear Klein-Gordon equation (El-sayed, 2003)

$$u_{tt} - u_{xx} = u \qquad \Omega = [0,1] \times [0,1]$$

With following conditions

$$u(x,0) = 1 + \sin(x) \qquad u_t(x,0) = 0$$

Whose exact solution is:

$$u(x,t) = \sin(x) + \cosh(t)$$

By using DQM we obtain

$$\frac{d^2u(xi,t)}{dt^2} = \left[a_{i1}^{(2)} \quad a_{i2}^{(2)} \ \dots\dots\dots \ a_{in}^{(2)}\right] u(x_j,t) + u(x_i,t)$$

And the initial condition $u(x_j, 0) = 1 + \sin(x_i)$, $\frac{du(x_j,0)}{dt} = 0$

This system solved by using finite difference to obtain:

Table (1): Approximate solution for example 1

x \ t	0.000	0.2000	0.4000	0.6000	0.8000	1.000
٠	١.٠٠٠٠	١.٠٢٠٠	١.٠٨٠٩	١.١٨٥١	١.٣٣٧٠	١.٥٤٣٠
٠.٠٩٥٥	١.٠٩٥٣	١.١١٥٣	١.١٧٦٢	١.٢٨٠٣	١.٤٣٢١	١.٦٣٧٧
٠.٣٤٥٥	١.٣٣٨٧	١.٣٥٨٧	١.٤١٩٥	١.٥٢٣٥	١.٦٧٥٠	١.٨٨٠٢
٠.٦٥٤٥	١.٦٠٨٨	١.٦٢٨٨	١.٦٨٩٦	١.٧٩٣٦	١.٩٤٥١	٢.١٥٠٥
٠.٩٠٤٥	١.٧٨٦١	١.٨٠٦١	١.٨٦٧٠	١.٩٧١٢	٢.١٢٣٢	٢.٣٢٩٨
١.٠٠٠٠	١.٨٤١٥	١.٨٦١٥	١.٩٢٢٤	٢.٠٢٦٧	٢.١٧٩٢	٢.٣٨٦٧

Table (2): exact solution for example 1

x \ t	0.000	0.2000	0.4000	0.6000	0.8000	1.000
٠	١.٠٠٠٠	١.٠٢٠١	١.٠٨١١	١.١٨٥٥	١.٣٣٧٤	١.٥٤٣١
٠.٠٩٥٥	١.٠٩٥٣	١.١١٥٤	١.١٧٦٤	١.٢٨٠٨	١.٤٣٢٨	١.٦٣٨٤
٠.٣٤٥٥	١.٣٣٨٧	١.٣٥٨٧	١.٤١٩٧	١.٥٢٤١	١.٦٧٦١	١.٨٨١٧
٠.٦٥٤٥	١.٦٠٨٨	١.٦٢٨٨	١.٦٨٩٨	١.٧٩٤٢	١.٩٤٦٢	٢.١٥١٩
٠.٩٠٤٥	١.٧٨٦١	١.٨٠٦٢	١.٨٦٧٢	١.٩٧١٦	٢.١٢٣٦	٢.٣٢٩٢
١.٠٠٠٠	١.٨٤١٥	١.٨٦١٥	١.٩٢٢٥	٢.٠٢٦٩	٢.١٧٨٩	٢.٣٨٤٦

Table (3): absolute error for example 1

x \ t	0.000	0.2000	0.4000	0.6000	0.8000	1.000
٠	٠.0000	5.9308e-05	٠.٠٠٠٠٢١١٤	٠.٠٠٠٠٣٧٥٣٩	٠.٠٠٠٠٤٠٤٩٢	0.000069803
٠.٠٩٥٥	٠.٠٠٠0	6.6146e-05	٠.٠٠٠٠٢٤٩٢٣	٠.٠٠٠٠٤٩٧٩٧	٠.٠٠٠٠٧٠٩٠١	٠.٠٠٠٠٦٩٨٠١
٠.٣٤٥٥	٠.0000	6.6909e-05	٠.٠٠٠٠٢٧٣٣	٠.٠٠٠٠٦٢٥٦٦	٠.٠٠٠١٠٩٨٦	٠.٠٠٠١٥٧٦١
٠.٦٥٤٥	٠.٠٠٠0	6.6781e-05	٠.٠٠٠٠٢٧٤١٦	٠.٠٠٠٠٦٢٤٣	٠.٠٠٠١٠٥٥٣	٠.٠٠٠١٣٣٤٧
٠.٩٠٤٥	٠.٠٠٠0	6.6153e-05	٠.٠٠٠٠٢٣٩٧٨	٠.٠٠٠٠٤١٧٩٦	٠.٠٠٠٠٣٣٠٩٨	٠.٠٠٠٠٦١٦٥٧
١.٠٠٠٠	٠.٠٠٠0	5.675e-05	٠.٠٠٠٠١٧٩٣٥	٠.٠٠٠٠١٩١٩٦	٠.٠٠٠٠٣١١٩٧	٠.٠٠٠٢١٥٣١

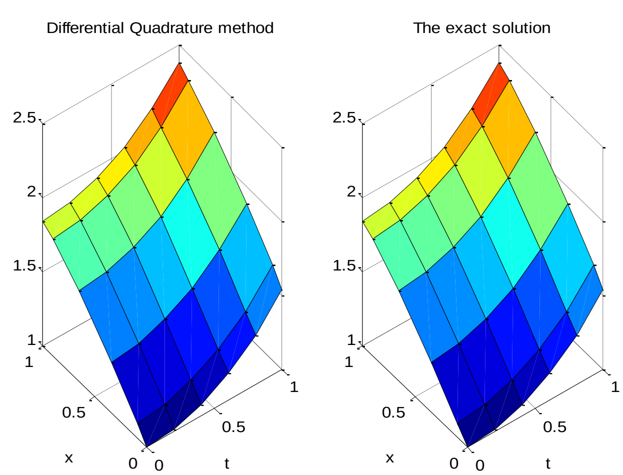

Fig. (1): Exact and approximate solution

Example 2:

Consider the nonlinear Klein-Gordon equation (Wazwaz, 2006) :

$$u_{tt} - u_{xx} + u^2 = 6xt(x^2 - t^2) + x^6t^6 \qquad \Omega = [0,1] \times [0,0.01]$$

With following conditions:

$u(x,0) = 0 \qquad u_t(x,0) = 0$

Whose exact solution is

$u(x,t) = x^3t^3$

By using DQM we obtain:

$$\frac{d^2u(x_i,t)}{dt^2} = \left[a_{i1}^{(2)} \quad a_{i2}^{(2)} \quad \dots \quad a_{in}^{(2)}\right] u(x_j,t) - (u(x_i,t))^2 + 6x_it\left(x_i^2 - t^2\right) + x_i^6t^6$$

And the initial condition $u(x_j,0) = 0$, $\frac{du(x_j,0)}{dt} = 0$

This system solved by using finite difference to obtain

Table (4): Approximate solution for example 2

x \ t	0.000	0.002	0.004	0.006	0.008	0.01
٠	٠.0000	٠.0000	٠.0000	3.3881e-027	3.3881e-027	٩.١٤٨-e-026
٠.٠٩٥٥	٠.٠٠٠0	٠.٠٠٠0	4.1778e-011	1.6711e-010	4.1778e-010	8.3555e-010
٠.٣٤٥٥	٠.0000	٠.0000	1.9794e-009	7.9177e-009	1.9794e-008	3.9588e-008
٠.٦٥٤٥	٠.٠٠٠0	٠.٠٠٠0	1.3458e-008	5.3832e-008	1.3458e-007	2.6916e-007
٠.٩٠٤٥	٠.٠٠٠0	٠.٠٠٠0	3.552e-008	1.4208e-007	3.552e-007	7.1041e-007
١.٠٠٠٠	٠.٠٠٠0	٠.٠٠٠0	4.8e-008	1.92e-007	4.8e-007	9.6e-007

Table (5): exact solution for example 2

x \ t	0.000	0.002	0.004	0.006	0.008	0.01
٠	٠.0000	٠.0000	٠.0000	٠.0000	٠.0000	٠.0000
٠.٠٩٥٥	٠.٠٠٠0	6.966e-012	5.5728e-011	1.8808e-010	4.4582e-010	8.7075e-010
٠.٣٤٥٥	٠.0000	3.2992e-010	2.6393e-009	8.9077e-009	2.1115e-008	4.1239e-008
٠.٦٥٤٥	٠.٠٠٠0	2.243e-009	1.7944e-008	6.0562e-008	1.4355e-007	2.8038e-007
٠.٩٠٤٥	٠.٠٠٠0	5.9201e-009	4.7361e-008	1.5984e-007	3.7889e-007	7.4001e-007
١.٠٠٠٠	٠.٠٠٠0	8e-009	6.4e-008	2.16e-007	5.12e-007	1e-006

Table (6): absolute error for example 2

x \ t	0.000	0.002	0.004	0.006	0.008	0.01
٠	٠.0000	٠.0000	٠.0000	3.3881e-027	3.3881e-027	9.148e-026
٠.٠٩٥٥	٠.٠٠٠0	6.966e-012	1.395e-011	2.0971e-011	2.8047e-011	3.5197e-011
٠.٣٤٥٥	٠.0000	3.2992e-010	6.599e-010	9.9001e-010	1.3203e-009	1.6509e-009
٠.٦٥٤٥	٠.٠٠٠0	2.243e-009	4.4862e-009	6.7296e-009	8.9734e-009	1.1218e-008
٠.٩٠٤٥	٠.٠٠٠0	5.9201e-009	1.184e-008	1.7761e-008	2.3682e-008	2.9604e-008
١.٠٠٠٠	٠.٠٠٠0	8e-009	1.6e-008	2.4001e-008	3.2002e-008	4.0004e-008

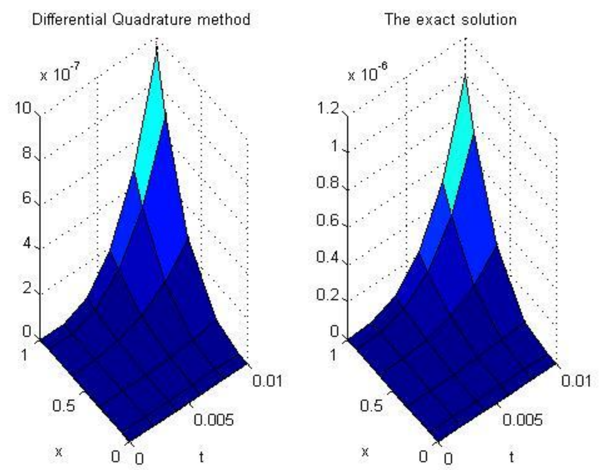

Fig. (2): Exact and approximate solution

Example 3:

Consider the linear Klein-Gordon equation (Deeba and Khuri, 1996):

$$u_{tt} - u_{xx} - 2u = -2\sin(x)\sin(t) \qquad \Omega = [0,1] \times [0,1]$$

With following conditions:

$u(x,0) = 0 \qquad u_t(x,0) = \sin(x)$

And exact solution:

$u(x,t) = \sin(x)\sin(\mathrm{t})$

By using DQM we obtain:

$$\frac{d^2u(x_i,t)}{dt^2} = \left[a_{i1}^{(2)} \quad a_{i2}^{(2)} \quad \dots\dots \quad a_{in}^{(2)}\right] u(x_j,t) + 2u(x_i,t) - 2\sin(x_i)\sin(t)$$

And the initial condition $u(x_j,0) = 0$, $\frac{du(x_j,0)}{dt} = \sin(x_i)$

This system solved by using finite difference to obtain:

Table (7): Approximate solution for example 3

x \ t	0.000	0.2000	0.4000	0.6000	0.8000	1.000
٠	٠.000	٠.000	0.0000029789	0.00002439	٠.٠٠٠١٠٦٦٣	٠.٠٠٠٣٣٧٧٦
٠.٠٩٥٥	٠.٠٠٠	٠.٠١٩٠٦٩	٠.٠٣٧٣٨٦	٠.٠٥٤٢٣٧	٠.٠٦٨٩٨٦	٠.٠٨١١٢١
٠.٣٤٥٥	٠.٠٠٠	٠.٠٦٧٧٣٢	٠.١٣٢٧٩	٠.١٩٢٦١	٠.٢٤٤٨٤	٠.٢٨٧٤٥
٠.٦٥٤٥	٠.٠٠٠	٠.١٢١٧٥	٠.٢٣٨٧	٠.٣٤٦٢٣	٠.٤٤٠١٢	٠.٥١٦٧١
٠.٩٠٤٥	٠.٠٠٠	٠.١٥٧٢٢	٠.٣٠٨٢٤	٠.٤٤٧١١	٠.٥٦٨٤٢	٠.٦٦٧٥٤
١.٠٠٠٠	٠.٠٠٠	٠.١٦٨٢٩	٠.٣٢٩٩٥	٠.٤٧٨٦٢	٠.٦٠٨٥٣	٠.٧١٤٨

Table (8): exact solution for example 3

x \ t	0.000	0.2000	0.4000	0.6000	0.8000	1.000
٠	٠.000	٠.000	٠.000	٠.000	٠.000	٠.000
٠.٠٩٥٥	٠.٠٠٠	٠.٠١٨٩٤٢	٠.٠٣٧١٣	٠.٠٥٣٨٣٧	٠.٠٦٨٣٩٧	٠.٠٨٠٢٣١
٠.٣٤٥٥	٠.٠٠٠	٠.٠٦٧٢٨١	٠.١٣١٨٨	٠.١٩١٢٢	٠.٢٤٢٩٤	٠.٢٨٤٩٧
٠.٦٥٤٥	٠.٠٠٠	٠.١٢٠٩٤	٠.٢٣٧٠٧	٠.٣٤٣٧٤	٠.٤٣٦٧	٠.٥١٢٢٦
٠.٩٠٤٥	٠.٠٠٠	٠.١٥٦١٨	٠.٣٠٦١٣	٠.٤٤٣٨٨	٠.٥٦٣٩٣	٠.٦٦١٥
١.٠٠٠٠	٠.٠٠٠	٠.١٦٧١٧	٠.٣٢٧٦٨	٠.٤٧٥١٣	٠.٦٠٣٦٣	٠.٧٠٨٠٧

Table (9): absolute error for example 3

x \ t	0.000	0.2000	0.4000	0.6000	0.8000	1.000
٠	٠.٠٠٠	٠.٠٠٠	0.0000029789	0.00002439	٠.٠٠٠١٠٦٦٣	٠.٠٠٠٣٣٧٧٦
٠.٠٩٥٥	٠.٠٠٠	٠.٠٠٠١٢٦٨٧	٠.٠٠٠٢٥٦٥٥	٠.٠٠٠٤٠٠٢٩	٠.٠٠٠٥٨٨٩٢	٠.٠٠٠٨٩٣٣
٠.٣٤٥٥	٠.٠٠٠	٠.٠٠٠٤٥٠٦٤	٠.٠٠٠٩١٠٢٩	٠.٠٠١٣٨٨٥	٠.٠٠١٩٠٠٥	٠.٠٠٢٤٨١٣
٠.٦٥٤٥	٠.٠٠٠	٠.٠٠٠٨١٠٠٧	٠.٠٠١٦٣٦٤	٠.٠٠٢٤٩٦	٠.٠٠٣٤١٤٦	٠.٠٠٤٤٥٢٣
٠.٩٠٤٥	٠.٠٠٠	٠.٠٠١٠٤٦١	٠.٠٠٢١١٣٤	٠.٠٠٣٢٣٧١	٠.٠٠٤٤٩١٦	٠.٠٠٦٠٤٣٢
١.٠٠٠٠	٠.٠٠٠	٠.٠٠١١١٩٧	٠.٠٠٢٢٦٦	٠.٠٠٣٤٨٨٣	٠.٠٠٤٨٩٦٣	٠.٠٠٦٧٢٨٢

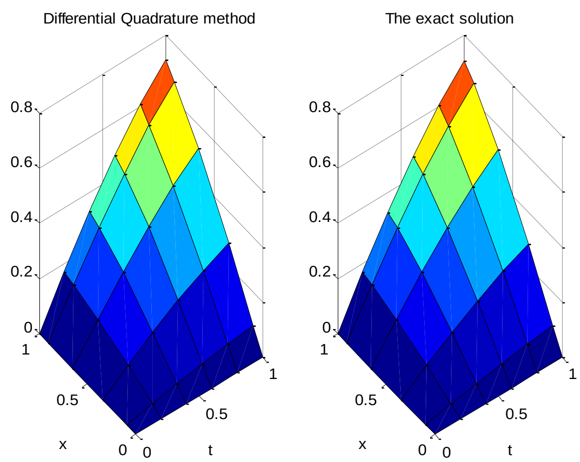

Fig. (3): Exact and approximate solution

Example 4:

Consider the nonlinear Klein-Gordon equation:

$$u_{tt} - u_{xx} + u^2 = -x cos t + x^2 cos^2 t \qquad \Omega = [0,1] \times [0,1]$$

With following conditions:

$$u(x,0) = 0 \qquad u_t(x,0) = 0$$

And exact solution:

$$u(x,t) = x cos(t)$$

By using DQM we obtain:

$$\frac{d^2 u(x_i,t)}{dt^2} = \left[a_{i1}^{(2)} \quad a_{i2}^{(2)} \ \dots \dots \quad a_{in}^{(2)}\right] u\big((x_j,t) - (u(x_i,t))^2 - x_i cost + {x_i}^2 cos^2 t$$

And the initial condition $u\big(x_j,0\big) = 0$, $\frac{du(x_j,0)}{dt} = 0$

This system solved by using finite difference to obtain:

Table (10): Approximate solution for example 4

x \ t	0.000	0.2000	0.4000	0.6000	0.8000	1.000
٠	٠.000	2.1316e-016	4.2633e-015	0.000062933	٠.٠٠٠٤٨٥٩٥	٠.٠٠٢٠٣٠٦
٠.٠٩٥٥	٠.٠٩٥٤٩٢	٠.٠٩٣٥٨٢	٠.٠٨٧٩٣٥	٠.٠٧٨٨٦١	٠.٠٦٧٠٤٥	٠.٠٥٣٧٥٤
٠.٣٤٥٥	٠.٣٤٥٤٩	٠.٣٣٨٥٨	٠.٣١٨٢٢	٠.٢٨٥٥٤	٠.٢٤٢٤٩	٠.١٩١٨٨
٠.٦٥٤٥	٠.٦٥٤٥١	٠.٦٤١٤٢	٠.٦٠٣٠١	٠.٥٤١٧٨	٠.٤٦١٧	٠.٣٦٧٨٥
٠.٩٠٤٥	٠.٩٠٤٥١	٠.٨٨٦٤٢	٠.٨٣٣٥١	٠.٧٤٩٧	٠.٦٤٠٩	٠.٥١٤٢٩
١.٠٠٠٠	١	٠.٩٨	٠.٩٢١٥٨	٠.٨٢٩٢٦	٠.٧٠٩٧٧	٠.٥٧١١٦

Table (11): exact solution for example 4

x \ t	0.000	0.2000	0.4000	0.6000	0.8000	1.000
٠	٠.000	٠.000	٠.000	٠.000	٠.000	٠.000
٠.٠٩٥٥	٠.٠٩٥٤٩٢	٠.٠٩٣٥٨٨	٠.٠٨٧٩٥٣	٠.٠٧٨٨١٣	٠.٠٦٦٥٣	٠.٠٥١٥٩٤
٠.٣٤٥٥	٠.٣٤٥٤٩	٠.٣٣٨٦	٠.٣١٨٢٢	٠.٢٨٥١٥	٠.٢٤٠٧١	٠.١٨٦٦٧
٠.٦٥٤٥	٠.٦٥٤٥١	٠.٦٤١٤٦	٠.٦٠٢٨٤	٠.٥٤٠١٩	٠.٤٥٦	٠.٣٥٣٦٣
٠.٩٠٤٥	٠.٩٠٤٥١	٠.٨٨٦٤٨	٠.٨٣٣١١	٠.٧٤٦٥٢	٠.٦٣٠١٨	٠.٤٨٨٧١
١.٠٠٠٠	١	٠.٩٨٠٠٧	٠.٩٢١٠٦	٠.٨٢٥٣٤	٠.٦٩٦٧١	٠.٥٤٠٣

Table (12): absolute error for example 4

x \ t	0.000	0.2000	0.4000	0.6000	0.8000	1.000
٠	٠.٠٠٠	2.1316e-016	4.2633e-015	6.2933e-005	٠.٠٠٠٤٨٥٩٥	٠.٠٠٢٠٣٠٦
٠.٠٩٥٥	٠.٠٠٠	6.3576e-06	1.8004e-005	4.8218e-005	٠.٠٠٠٥١٤٩٨	٠.٠٠٢١٥٩٤
٠.٣٤٥٥	٠.٠٠٠	2.3002e-05	2.8083e-006	٠.٠٠٠٣٩٦٢٦	٠.٠٠١٧٨٦٣	٠.٠٠٥٢٠٦١
٠.٦٥٤٥	٠.٠٠٠	4.3576e-05	٠.٠٠٠١٦٤٤٣	٠.٠٠١٥٩٣	٠.٠٠٥٧٠١٩	٠.٠١٤٢١٩
٠.٩٠٤٥	٠.٠٠٠	6.022e-05	٠.٠٠٠٤٠٥١٢	٠.٠٠٣١٧٤٨	٠.٠١٠٧٢	٠.٠٢٥٥٨٥
١.٠٠٠٠	٠.٠٠٠	6.6578e-05	٠.٠٠٠٥٢٣٠١	٠.٠٠٣٩٢٢٦	٠.٠١٣٠٦٥	٠.٠٣٠٨٥٦

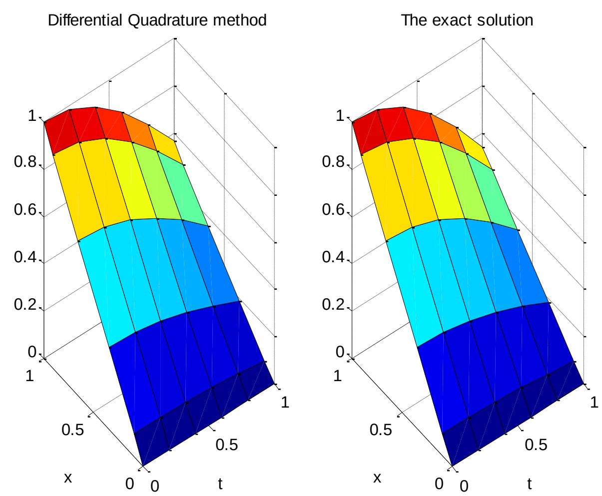

Fig. (4): Exact and approximate solution

Conclusion

In this work, we calculated the approximation solution of the Klein –Gordon equation by using the (DQM),we solve several example of linear and nonlinear equation , our results show near the approximate solution to the exact solution which means important ,powerful and efficient method for solve this equation .

Reference

Bellman, R. and Casti, J., "Differential Quadrature and Long-Term Integration," Journal of Mathematical Analysis and Application, Vol. 34, pp. 235_238 (1971).

Bellman, R. E., Kashef, B. G. and Casti, J., "Differential Quadrature: A Technique for Rapid Solution of Nonlinear Partial Differential

Equations," Journal Computational Physics, Vol. 10, pp. 40_52 (1972).

Bert, C.W.and Malik., "differential Quadrature Method in computational mechanics: A Review, "Applied Mechanics Review, Vol.49, pp.1-28(1996).

Chen, W. L., Striz, A. G. and Bert, C.W., "High-Accuracy Plane Stress and Plate Elements in the Quadrature Element Method," International Journal of Solid and Structures, Vol. 37, pp. 627_647 (2000).

Dodd, R.K., I.C. Eilbeck, J.D Gibbon, 1982. Solitons and nonlinear wave equations. London: Academic.

Deeba, E.Y., S.A. Khuri, 1996. A decomposition method for solving the nonlinear Klein-Gordon equation, J. Comp. Phys., 124: 442-448.

Du, H., Liew, K.M. and Lim, M. K., "Generalized Differential Quadrature Method for Buckling Analysis," Journal of Engineering Mechanics, Vol. 122, pp. 95- 100 (1996).

Feng, Y. and Bert, C. W., "Application of the Quadrature Method to Flexural Vibr-ation Analysis of a Geometrically Nonlinear Beam," Nonlinear Dynamics Vol. 3, pp. 13_18 (1992).

Han, J. B. and Liew, K. M., "Axisymmetric Free Vibration of Thick Annular Plates," International Journal of Mechanical Science, Vol. 41, pp. 1089_1109 (1999).

Inc., M., 2006. New compaction and solitary pattern solutions of the nonlinear modified dispersive Klein Gordon equations, Chaos, Solitons & Fractals, doi: 10.1016/chaos., 01.083.

Liew, K. M., Han, J. B. and Xiao, Z. M., "Differentia Quadrature Method for Thick Symmetric Cross-Ply Laminates with First-Order Shear Flexibility," International Journal of Solids and Structures, Vol. 33, pp. 2647-2658 (1996).

Wang, X.Y., 1988. Exact and explicit solitary wave solutions for the generalized Fisher equation, J. Phys Lett. A 131(4/5):277-279.

Whitham, G.B., Linear and nonlinear waves, Wiley, New York, (1974).

Wazwaz, A.M., 2006. The variational iteration method for rational solutions for KdV, K (2,2), Burgers, and cubic Boussinesq equations, J. Comput. Appl. Math. doi: 10.1016/j.cam. 07: 010.