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NUMERICAL SOLUTIONS OF THE GENERALIZED BURGERS – HUXLEY EQUATION BY FINITE DIFFERENCE METHOD

Zenab k.jabar

Math.Dept.,Computer and Math.coll.,Thi-Qar univ.

Abstract

In this paper, numerical solutions of the generalized Burgers-Huxley equation are obtained by using explicit finite difference method and Crank- Nicalson(C.N) finite difference method. we compare our result with the exact solution ,our Numerical results show that C.N finite difference method is more efficient and more accurate than the explicit finite difference method when the value of δ is small.

Keywords: finite difference, crank-Nicalson method, explicit method

1.Introduction

Nonlinear partial differential equations are encountered in various fields of science .Generalized Burgers-Huxley[1] equation being nonlinear partial is of high importance for describing the interaction between reaction mechanisms, convection effects and diffusion transports, since there exists no general technique for finding analytical solutions of nonlinear diffusion equation so far , numerical solutions of nonlinear differential equation are of great importance in physical problems, there are many researchers who used various numerical techniques to obtain numerical solution of the Burgers-Huxley equation. Wang et al.[2]studied the solitary wave solutions of the generalized Burgers-Huxley equation and Estevez [3] present non-classical symmetries and the singular modified Burgers and Burgers-Huxley equation .in the past few years, various powerful mathematical methods such as spectral methods[4-6],Adomian decomposition method[7-9],homotopy analysis method[10],the tanh-coth method[11],variational iteration method[12,13]and Hopf-Cole transformation[14] have been used in attempting to solve the equation.

The generalized Beurgrs-Huxley equation problem arises in various fields of science are [1]

$$\frac{\partial u}{\partial t}+\alpha u^{\delta}\frac{\partial u}{\partial x}-\frac{\partial^2 u}{\partial x^2}=bu(1-u^{\delta})(u^{\delta}-a) \qquad 0\le x\le 1 \ ,t\ge 0 \qquad (1)$$

With the initial condition

$$u(x,0)=(\frac{a}{2}+\frac{a}{2}\tanh(a_1 x))^{\frac{1}{\delta}} \qquad (2)$$

and the boundary conditions

$$u(0,t) = (\frac{a}{2} + \frac{a}{2}\tanh(-a_1 a_2 t))^{\frac{1}{\delta}} \qquad t \geq 0 \qquad (3)$$

$$u(1,t) = (\frac{a}{2} + \frac{a}{2}\tanh[a_1(1-a_2 t)])^{\frac{1}{\delta}} \qquad t \geq 0 \qquad (4)$$

exact solution of Eq.(1) is The

$$u(x,t) = (\frac{a}{2} + \frac{a}{2}\tanh[a_1(x-a_2 t)])^{\frac{1}{\delta}} \qquad t \geq 0 \qquad (5)$$

where

$$a_1 = \frac{-\alpha\delta + \delta\sqrt{\alpha^2 + 4b(1+\delta)}}{4(1+\delta)}a \qquad (6a)$$

$$a_2 = \frac{a\alpha}{1+\delta} - \frac{(1+\delta-a)(-\alpha+\sqrt{\alpha^2+4b(1+\delta)})}{2(1+\delta)} \qquad (6b)$$

Here α,b,a and δ are parameters that b≥0 ,δ>0 .The role of the parameters on exact solutions was analyzed by Yefimova and kudryashov[14].if b=0,Eq.(1) reduces to the Burgers Eq.,when α=0 ,it is the Fitzhugh-Nagoma Eq.[15,16]

The present results are compared with exact solution to verify the effectiveness of the current method for different value of δ.

2.prosent model

2.1 Explicit Finite Difference Method

The first step is to choose integers h and k such that N=(b-a)/h and m=(d-c)/ k

Partition the interval **[a,b]** into N equal parts of width h and the interval [c,d] then the mesh point (x_0, y_0) into m equal part of width k, if the origin **With** x_i

and the mesh point

Using Ta $x_i = x_0 + ih \quad 0 \leq i \leq N$ he forward approximatior t_j **With** $tj = y_0 + jk \quad 0 \leq j \leq m$

$$\frac{\partial u}{\partial t} = \frac{u_{i,j+1} - u_{i,j}}{k} \qquad (7)$$

the forward approximation for x

$$\frac{\partial u}{\partial x} = \frac{u_{i+1,j} - u_{i,j}}{h} \qquad (8)$$

and the centered difference approximation

$$\frac{\partial^2 u}{\partial x^2} = \frac{(u_{i+1,j} - 2u_{i,j} + u_{i-1,j})}{h^2} \qquad (9)$$

Substituting (7), (8) and (9) into equation (1) we get

$$\frac{u_{i,j+1}-u_{i,j}}{k}+\alpha u_{i,j}^{\delta}\left(\frac{u_{i+1,j}-u_{i,j}}{h}\right)-\left(\frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{h^2}\right)=bu_{i,j}(1-u_{i,j}^{\delta})(u_{i,j}^{\delta}-a) \qquad (10)$$

$$\Rightarrow \frac{u_{i,j+1}-u_{i,j}}{k}=-\alpha u_{i,j}^{\delta}\left(\frac{u_{i+1,j}-u_{i,j}}{h}\right)+\left(\frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{h^2}\right)+bu_{i,j}(1-u_{i,j}^{\delta})(u_{i,j}^{\delta}-a) \qquad (11)$$

$let \quad r=k/h^2$

$$u_{i,j+1}-u_{i,j}=-\alpha hru_{i,j}^{\delta}u_{i+1,j}+\alpha hru_{i,j}^{\delta}u_{i,j}+ru_{i+1,j}-2ru_{i,j}+ru_{i-1,j}+kb(u_{i,j}^{\delta+1}-au_{i,j}-u_{i,j}^{2\delta+1}-au_{i,j}^{\delta+1}) \qquad (12)$$

$$\Rightarrow u_{i,j+1}=(1-2r+\lambda\alpha hru_{i,j}^{\delta}+kbu_{i,j}^{\delta}-akb-kbu_{i,j}^{2\delta}-kbau_{i,j}^{\delta})u_{i,j}+(r-\alpha hru_{i,j}^{\delta})u_{i+1,j}+ru_{i-1,j} \qquad (13)$$

$$\Rightarrow u_{i,j+1}=ru_{i-1,j}+(1-2r+\alpha hru_{i,j}^{\delta}+kbu_{i,j}^{\delta}-akb-kbu_{i,j}^{2\delta}-kbau_{i,j}^{\delta})u_{i,j}+(r-\alpha hru_{i,j}^{\delta})u_{i+1,j} \qquad (14)$$

Eq.(14) is the approximated finite difference by using explicit scheme for Burgers-Huxley equation ,we can calculate the row (j+1) from the known value of row(j).

2.2 Crank-Nicholson Finite Difference Method

In this method we use central difference in time j and j+1

$$\frac{\partial^2 u}{\partial x^2}=\frac{1}{2}\left(\frac{u_{i-1,j+1}-2u_{i,j+1}+u_{i+1,j+1}+u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{h^2}\right) \qquad (15)$$

the forward finite difference about x in time j and j+1 is

$$\frac{\partial u}{\partial x}=\frac{1}{4}\left(\frac{u_{i+1,j+1}-u_{i-1,j+1}+u_{i+1,j}-u_{i-1,j}}{h}\right) \qquad (16)$$

the forward finite difference about t in time j and j+1 is

$$\frac{\partial u}{\partial t}=\frac{u_{i,j+1}-u_{i,j}}{k} \qquad (17)$$

Substituting Eqs.(15-17) into (1) we can get

$$\frac{u_{i,j+1}-u_{i,j}}{k}-\frac{1}{2}\left(\frac{u_{i-1,j+1}-2u_{i,j+1}+u_{i+1,j+1}+u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{h^2}\right)$$
$$+\alpha u_{i,j}^{\delta}\left(\frac{1}{4}\left(\frac{u_{i+1,j+1}-u_{i-1,j+1}+u_{i+1,j}-u_{i-1,j}}{h}\right)\right)=bu_{i,j}(1-u_{i,j}^{\delta})(u_{i,j}^{\delta}-a) \qquad (18)$$

$$\Rightarrow \frac{u_{i,j+1}-u_{i,j}}{k}+\alpha\frac{u_{i,j}^{\delta}}{4h}(u_{i+1,j+1}-u_{i-1,j+1})+\alpha\frac{u_{i,j}^{\delta}}{4h}(\quad u_{i+1,j}-u_{i-1,j})-\frac{1}{2h^2}(u_{i-1,j+1}-2u_{i,j+1}+$$
$$u_{i+1,j+1})-\frac{1}{2h^2}(u_{i-1,j}-2u_{i,j}+u_{i+1,j})=b(u_{i,j}-(u_{i,j})^{\delta+1})(u_{i,j}^{\delta}-a) \qquad (19)$$

$$\Rightarrow 2u_{i,j+1}-2u_{i,j}+\alpha\frac{hru_{i,j}^{\delta}}{2}(u_{i+1,j+1}-u_{i-1,j+1})+\alpha\frac{hru_{i,j}^{\delta}}{2}(u_{i+1,j}-u_{i-1,j})-r(u_{i-1,j+1}-2u_{i,j+1}$$
$$+u_{i+1,j+1})-r(u_{i-1,j}-2u_{i,j}+u_{i+1,j})=2kb(u_{i,j}-(u_{i,j})^{\delta+1})(u_{i,j}^{\delta}-a) \qquad (20)$$

$$\Rightarrow 2u_{i,j+1}+\alpha\frac{rh}{2}u_{i,j}^{\delta}(u_{i+1,j+1}-u_{i-1,j+1})-r(u_{i-1,j+1}-2u_{i,j+1}+u_{i+1,j+1})=2u_{i,j}-$$
$$\alpha\frac{hr}{2}u_{i,j}^{\delta}(u_{i+1,j}-u_{i-1,j})+r(u_{i-1,j}-2u_{i,j}+u_{i+1,j})+2kb(u_{i,j})^{\delta+1}-2kb(u_{i,j})^{2\delta+1}$$
$$-2kabu_{i,j}+2kab(u_{i,j})^{\delta+1} \qquad (21)$$

$$\Rightarrow (-r-\alpha\frac{hr}{2}u_{i,j}^{\delta})u_{i-1,j+1}+(2+2r)u_{i,j+1}+(-r+\alpha\frac{hr}{2}u_{i,j}^{\delta})u_{i+1,j+1}=(\alpha\frac{hr}{2}u_{i,j}^{\delta}+$$
$$r)u_{i-1,j}+(-\alpha\frac{hr}{2}u_{i,j}^{\delta}+r)u_{i+1,j}+(2-2r)u_{i,j}+2kb(u_{i,j})^{\delta+1}-2kb(u_{i,j})^{2\delta+1}-$$
$$2kab(u_{i,j})+2kab(u_{i,j})^{\delta+1} \qquad (22)$$

Eq.(22) represent the approximated finite difference that can be obtained by using C.N, we observe that the above Eq.can be written as Ax=b where Ax is

$$\begin{bmatrix} 2+2r & (-r+r\frac{h\alpha}{2}u_{1,j}^{\delta}) & 0 & \cdots & 0 \\ -r-r\frac{h\alpha}{2}u_{2,j}^{\delta} & 2+2r & -r+r\frac{h\alpha}{2}u_{2,j}^{\delta} & \ddots & 0 \\ 0 & -r-r\frac{h\alpha}{2}u_{2,j}^{\delta} & 2+2r & \ddots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots \\ 0 & \cdots & 0 & \ddots & 2+2r \end{bmatrix}\begin{bmatrix} u_{2,j+1} \\ u_{3,j+1} \\ u_{4,j+1} \\ \vdots \\ \\ \vdots \\ u_{15,j+1} \end{bmatrix}$$

and b is

$$\begin{bmatrix} 2r+rhau_{2,j}^{\delta}(u_{1,j+1}+u_{1,j}+(2+2r)u_{2,j}+(r-\frac{rha}{2}u_{3,j}+2kbu_{2,j}^{\delta}-u_{2,j}^{2\delta+1}-au_{2,j}+au_{2,j}^{\delta+1} \\ r+\frac{rha}{2}u_{3,j}^{\delta}u_{2,j}+(2+2r)u_{3,j}+r-\frac{rha}{2}u_{3,j}^{\delta}u_{4,j}+2kbu_{3,j}^{\delta+1}-u_{3,j+1}^{2d+1}-au_{3,j}+au_{3,j}^{\delta+1} \\ r+\frac{rha}{2}u_{4,j}^{\delta}u_{3,j}+(2+2r)u_{4,j}+r-\frac{rha}{2}u_{4,j}^{\delta}u_{5,j}+2kbu_{4,j}^{\delta+1}-u_{4,j+1}^{2d+1}-au_{4,j}+au_{4,j}^{\delta+1} \\ \vdots \\ \vdots \\ \frac{rha}{2}(u_{16,j}^{\delta}+r)u_{15,j}+(2+2r)u_{16,j}+(2r-rhau_{17,j}^{\delta})(u_{17,j+1}+u_{17,j}) \end{bmatrix}$$

4.Numerical Results

To solve Eq.(1) numerically by using FDM,we are used two methods the first is an explicit method and the second is (C.N),there for we compare difference between the exact and approximated solutions and increase the accuracy of solutions at this equation by using (C.N) method.

Consider the generalized Burgers-Huxley equation in the form (1) with initial condition (2) and boundary condition (3),(4) and the exact solution (5) . The results are compared with the exact solution, absolute error for different values of a ,b ,c and δ is reported which is defined by |u exact –u approximate|

We use four case for different value of δ .All results are computed by using matlab 6.5 applied on pentium4 computer ,N,k and tn are taken to be 16,0.0001 and 0.2 respectively.

Special case

Case 1

In table (1) the absolute error are shown for δ=1,α=1,b=1,and a=0.001.From this table we note that the error of (C.N) is less than that of explicit for δ=1.

Case 2

In table (2) the absolute error are shown for δ=2,α=0.1,b=0.001,and a=0.0001.From this table we note that the error of (C.N) is less than that of explicit for δ=2.

Case 3

In table (3) the absolute error are shown for δ=6,α=1,b=1,a=0.001 .From this table we note that the error of C.N is equal to that of explicit method for δ=6

Case 4

In table (4) the absolute error are shown for δ=8,α=5,b=10,a=0.001 .From this table we note that the error of C.N is equal to that of explicit method for δ=8.

From case 3 and case 4 we can observed that when the value of δ is large then the error is increase and the two method gives the same results ,we conclude if we take small value of δ then the error is decrease and the C.N method is more efficient and more accurate than the explicit method.

Table 1: The absolute error computed by C.N and explicit methods with (δ=1)

Exact value	Approximated explicit	Approximated C.N	Error explicit	Error C.N
1.0e-003 *	1.0e-003*	1.0e-003*	1.0e004*	1.0e-007*
0.5000	0.5000	0.5000	0	0.2497
0.5000	0.4879	0.5000	0.1217	0.2497
0.5000	0.4776	0.5000	0.2248	0.2497
0.5000	0.4690	0.5000	0.3104	0.2497
0.5001	0.4621	0.5000	0.3792	0.2497
0.5001	0.4569	0.5000	0.4320	0.2497
0.5001	0.4531	0.5000	0.4693	0.2497
0.5001	0.4509	0.5001	0.4915	0.2497
0.5001	0.4502	0.5001	0.4989	0.2497
0.5001	0.4509	0.5001	0.4916	0.2497
0.5001	0.4532	0.5001	0.4694	0.2497
0.5001	0.4569	0.5001	0.4321	0.2497
0.5001	0.4622	0.5001	0.3793	0.2497
0.5001	0.4691	0.5001	0.3104	0.2497
0.5001	0.4776	0.5001	0.2248	0.2497
0.5001	0.4880	0.5001	0.1217	0.2497
0.5001	0.5001	0.5001	0	0.2497

Table 2: The absolute error computed by C.N and explicit methods with (δ=2)

Exact value	Approximated explicit	Approximated C.N	Error explicit	Error C.N
.				1.0e-005*
0.1495	0.1495	0.1495	0	0.1722
0.1495	0.1481	0.1495	0.0015	0.1722
0.1495	0.1468	0.1495	0.0027	0.1722
0.1495	0.1458	0.1495	0.0038	0.1722
0.1495	0.1449	0.1495	0.0046	0.1722
0.1495	0.1443	0.1495	0.0053	0.1722
0.1495	0.1438	0.1495	0.0057	0.1722
0.1495	0.1435	0.1495	0.0060	0.1722
0.1495	0.1434	0.1495	0.0061	0.1722
0.1495	0.1435	0.1495	0.0060	0.1722
0.1495	0.1438	0.1495	0.0057	0.1722
0.1495	0.1443	0.1495	0.0053	0.1722
0.1495	0.1449	0.1495	0.0046	0.1722
0.1495	0.1458	0.1495	0.0038	0.1722
0.1495	0.1468	0.1495	0.0027	0.1722
0.1495	0.1481	0.1495	0.0015	0.1722
0.1495	0.1495	0.1495	0	0.1722

Table 3: The absolute error computed by C.N and explicit methods with (δ=6)

Exact value	Approximated explicit	Approximated C.N	Error explicit	Error C.N
.				
0.2817	0.2817	0.2817	0	0
0.2817	0.2498	0.2498	0.0320	0.0320
0.2818	0.2236	0.2236	0.0581	0.0581
0.2818	0.2026	0.2026	0.0792	0.0792
0.2818	0.1860	0.1860	0.0957	0.0957
0.2818	0.1736	0.1736	0.1081	0.1081
0.2818	0.1650	0.1650	0.1167	0.1167
0.2818	0.1599	0.1599	0.1218	0.1218
0.2818	0.1583	0.1583	0.1235	0.1235
0.2818	0.1599	0.1599	0.1218	0.1218
0.2818	0.1650	0.1650	0.1167	0.1167
0.2818	0.1737	0.1737	0.1081	0.1081
0.2818	0.1861	0.1861	0.0957	0.0957
0.2818	0.2026	0.2026	0.0792	0.0792
0.2818	0.2236	0.2236	0.0581	0.0581
0.2818	0.2498	0.2498	0.0320	0.0320
0.2818	0.2818	0.2818	0	0

Table 4: The absolute error computed by C.N and explicit methods with (δ=8)

Exact value	Approximated explicit	Approximated C.N	Error explicit	Err G.N
0.2900	0.2900	0.2900	0	0
0.2900	0.1670	0.2900	0	0.1230
0.2900	0.0963	0.2900	0.1230	0.1937
0.2900	0.0556	0.2900	0.1937	0.2344
0.2900	0.0323	0.2900	0.2344	0.2577
0.2900	0.0190	0.2900	0.2577	0.2710
0.2900	0.0117	0.2900	0.2710	0.2783
0.2900	0.0081	0.2900	0.2783	0.2819
0.2900	0.0070	0.2900	0.2819	0.2830
0.2900	0.0081	0.2900	0.2830	0.2819
0.2900	0.0117	0.2900	0.2819	0.2783
0.2900	0.0190	0.2900	0.2783	0.2710
0.2900	0.0323	0.2900	0.2710	0.2577
0.2900	0.0556	0.2900	0.2577	0.2344
0.2900	0.0963	0.2900	0.2344	0.1937
0.2900	0.1670	0.2900	0.1937	0.1230
0.2900	0.2900	0.2900	0.1230	0
.			0	

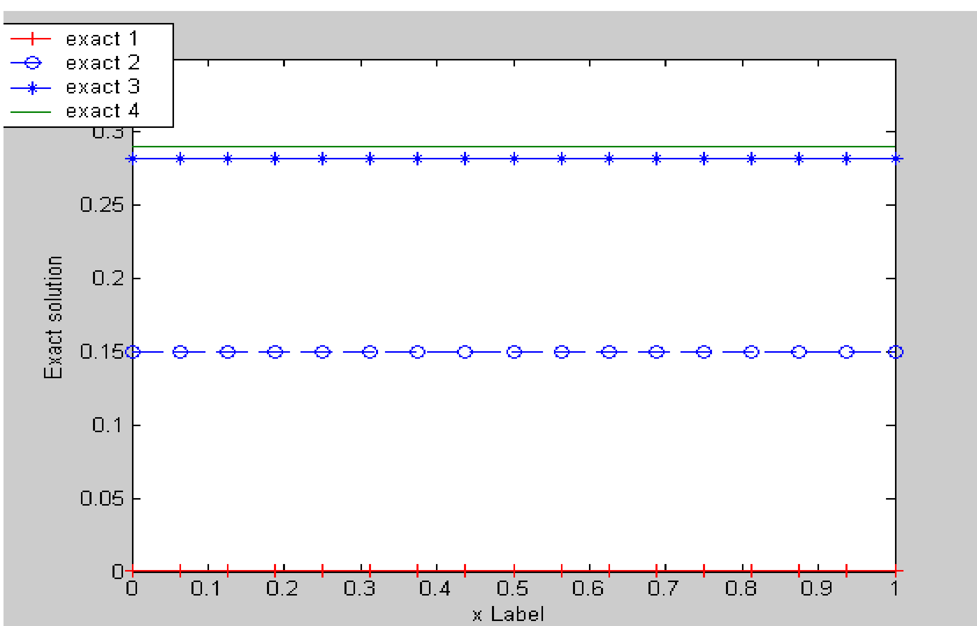

Fig.(1): Exact solution for Beurgrs-Huxley equation with (δ=1, δ=2, δ=6, δ=8)

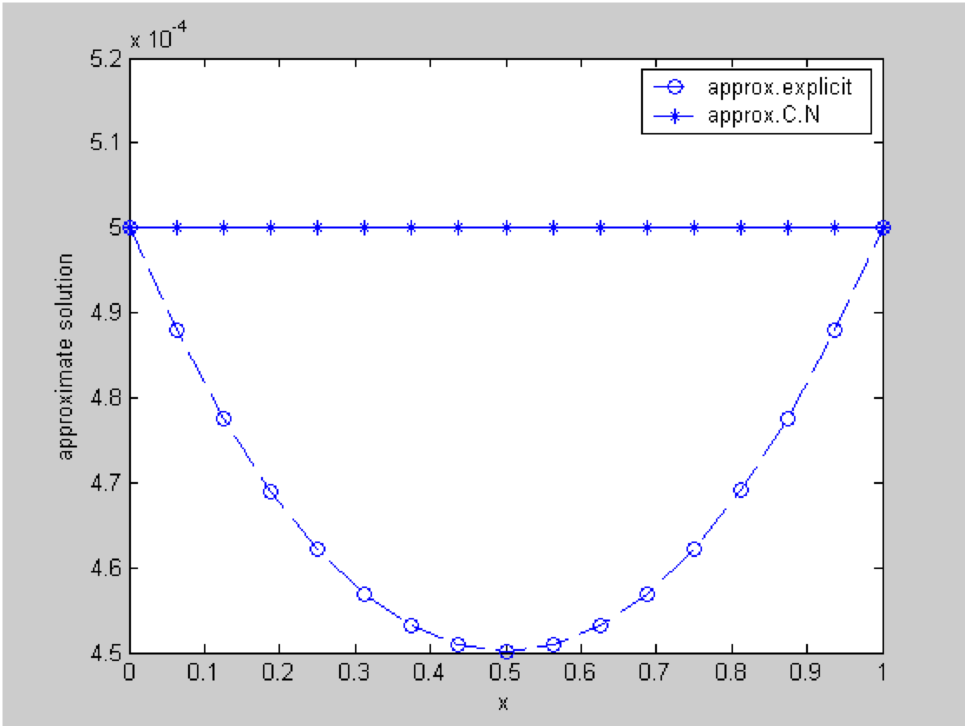

Fig.(2) Approximate solution by C.N and explicit with (δ=1)

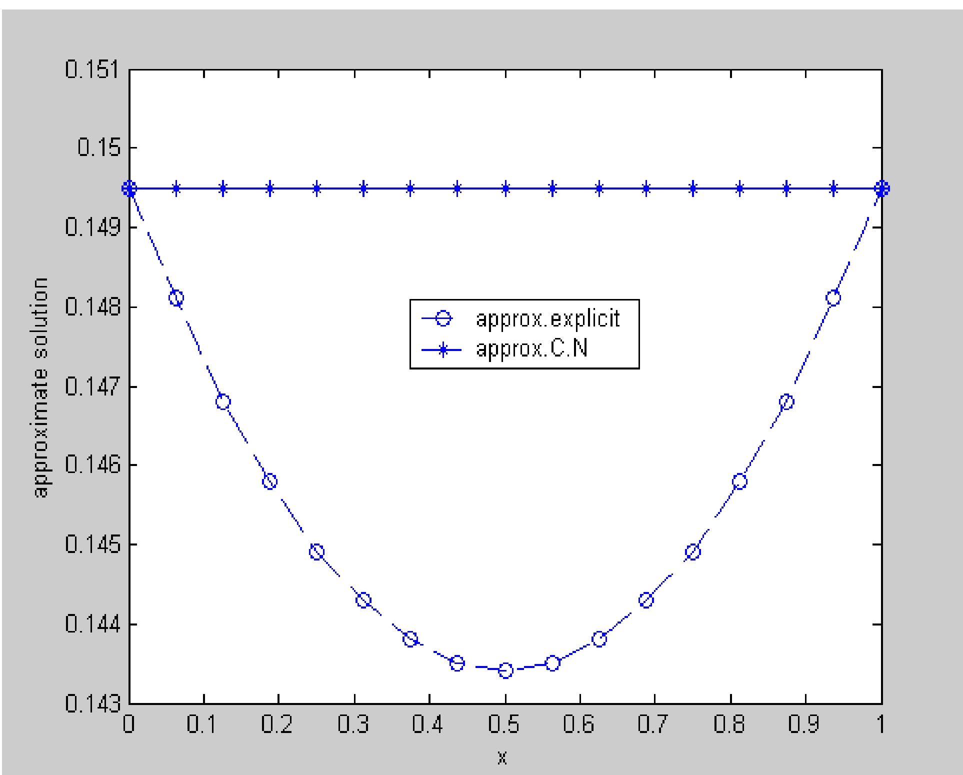

Fig.(3)Approximate solution by C.N and explicit with (δ=2)

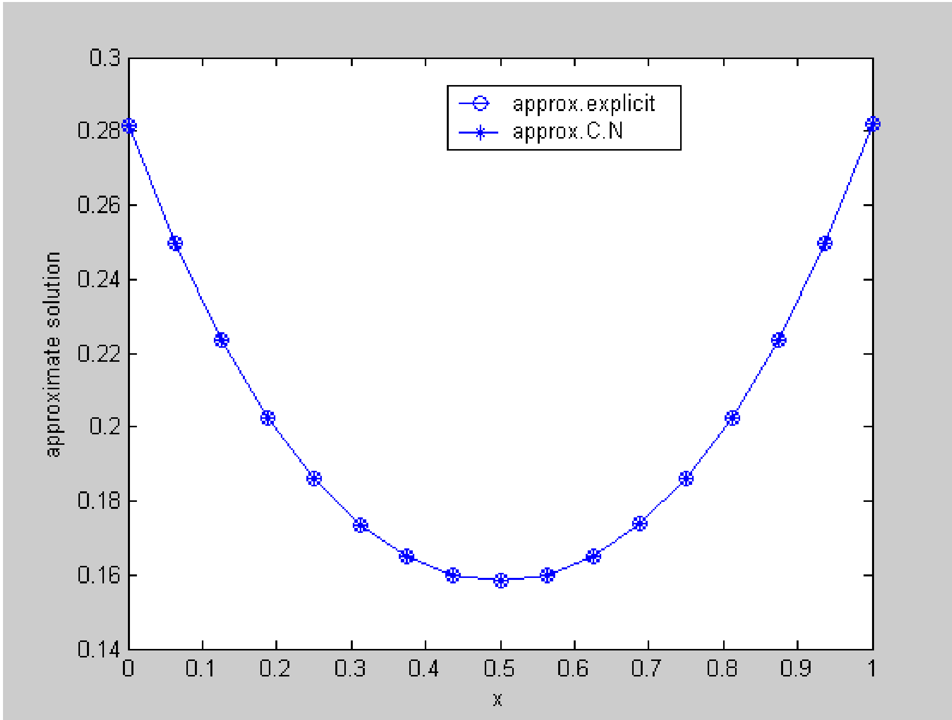

Fig.(4) Approximate solution by C.N and explicit with (δ=6)

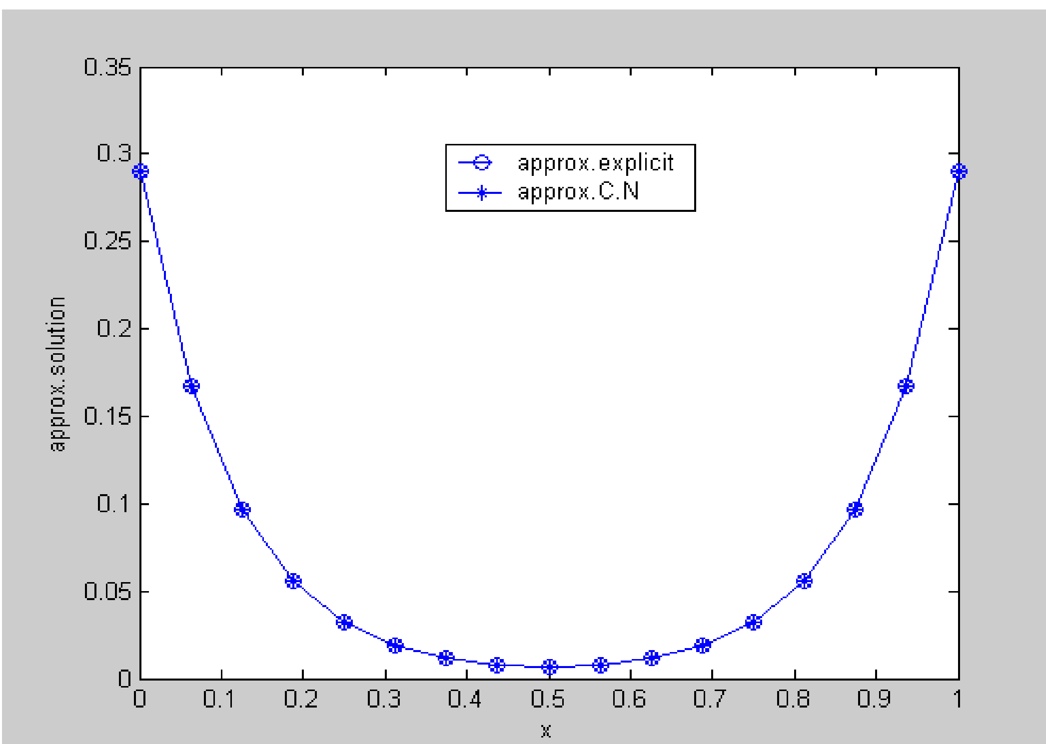

Fig.(5) Approximate solution by C.N and explicit with (δ=8)

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قسم الرياضيات - كلية علوم الحاسبات والرياضيات - جامعة ذي قار

الخلاصة

تناولنا في هذا البحث حل معادلة بورغر-هوكسلي المعممة باستخدام طريقة الفرو قات المنتهية إذ تم استخدام طريقتين الأولى هي الطريقة الصحيحة والثانية هي طريقة كرانك –نيكالسون كذلك ناقشنا الخطأ الناتج من هذه الطرق بالمقارنة مع الحل الحقيقي وأظهرت النتائج أن طريقة كرانك –نيكالسون هي أكثر كفاءة وأكثر دقة من الطريقة الصحيحة عندما تكون قيم δ صغيرة.