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website: http://jsci.utq.edu.iq Email: utjsci@utq.edu.iq

The Finite Element Method for Nonlinear Huxely Equation

Ahmed Jabbar Hussein

Department of Mathematics - College of Education - University of Thi-Qar

Abstract

In this paper , we present a finite element method (F.E.M.) for solving the non-linear huxely equation by using Crank-Nicolson scheme with the predictor - corrector method. The numerical results and absolute errors showed that the Crank - Nicolson finite element method is more accurate and better than respectively from the numerical results and the absolute errors which that presented by Explicit and Crank-Nicolson finite difference method (F.D.M.) [7] .

Keyword: Finite element, Huxely equation, Crank-Nicolson

Introduction

From nonlinear diffusion equations the Generalized burgers huxely equation

$$\frac{\partial u}{\partial t}+\alpha u^{\delta}\frac{\partial u}{\partial x}-\frac{\partial^2 u}{\partial x^2}=\beta u(1-u^{\delta})(u^{\delta}-a) \qquad (1)$$

$$\alpha \ge 0,\ \beta \ge 0,\ \delta > 0,\ and\ \ a \in (0,1)$$

Equation (1) is extension formula for burgers huxely – equation. When $\alpha \neq 0,\ \beta \neq 0\ and\ \delta = 1$, The above equation transform to burgers huxely equation:

$$\frac{\partial u}{\partial t}+\alpha u\frac{\partial u}{\partial x}-\frac{\partial^2 u}{\partial x^2}=\beta u(1-u)(u-a)\ . \qquad (2)$$

Equation (2) is prime model to describe the relation between the reaction procedure and effectives the solution and diffusion. Equation (1) has special two cases are burgers and huxely equation, When $\delta = 1\ and\ \beta = 0$ equation (1) transform to the burgers equation

$$\frac{\partial u}{\partial t}+\alpha u\frac{\partial u}{\partial x}-\frac{\partial^2 u}{\partial x^2}=0 \qquad (3)$$

Which describe the far domain to wave diffusion in dissipative dynamical systems, When $\delta = 1\ and\ \alpha = 0$ equation (1) transform to huxely equation

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = \beta u(1-u)(u-a) \quad in\ \Omega \times [0,\infty) \tag{4}$$

$0 < a < 1,\ \beta > 0,\ -1 \le x \le 1,\ and\ \ t \ge 0$

with inital condition and boundary condition

$u(x,0) = (b-H)x^2 + H$

$0 \le b \le 1,\ \ and\ \ 0 < H \le 1,$

$u(-1,t) = u(1,t) = b$

Which describe the diffusion the nervous impulses in nervous fibers and wall motion in liquid crystals .The effect of constant H on the numerical solution is made the solution curve either convex (the maximum value to solution at x=0) or concave (the minimum value to the solution at x=0).

Equations (3) and (4) have important rules in nonlinear physical, because these equations important in the nonlinear phenomena's study .

Frank and Zeldovich [3] (1938) introduced equation (4) as model to study diffusion the nervous impulses, they proved that the transformation wave has the figure and the Velocity as follows :

$$u(x,t) = \frac{1}{1+\exp[(x-vt)/\sqrt{2}]}, \qquad v = \frac{1-2a}{\sqrt{2}}$$

Maginu [4] (1978) studied the stability of steady state solutions for this type from the diffusion equations. He using the Liapunov method to find the stability condition and he show that this condition connected with Existence condition for these solutions.

Manorenjan [6] (1984) studied in detail the steady state case $u = a$, he used the pseudo-spectral method and Fortran IV language for this proposed .

Binczak, Eilbeck and Scort [1] (2001) using this equation as model for operating nervous transducer (conducted), they using the numerical results (by using predictor-corrector method) to expression this operation.

Mohammad abd [7] (2005) studied stationary stability and numerical solution for this equation by using stability analysis type Fourier (von Neumann). Also he solve this equation numerically in two methods from finite difference methods, the first is the Explicit finite difference method, the second is the Crank – Nicolson finite difference method.

In this paper, huxely equation was solved by the Crank – Nicolson finite element method with predictor - corrector method .The numerical results are compared with the numerical results [7]

The stationary solution for Huxely equation (4) is (see [5])

$$u(x)=\begin{cases} 3a\Big/\sqrt{(2-a)(1/2-a)}\cosh(\sqrt{a}x)+1+a & 0<a<1/2,\quad \beta=1 \\ \frac{1}{2}+\alpha\, sn\left(\frac{x}{\sqrt{2}}(1/2-\alpha^2)^{1/2},\ \alpha\Big/(1/2-\alpha^2)^{1/2}\right), & a=1/2\ ,\ \beta=1,\quad 0<\alpha<1 \end{cases} \tag{5}$$

Where $sn(v,d)$ represent the Jacobi Elliptic Function for argument v and the modulus d , The Taylor series of function $sn(v,d)$ is (see [2]) :

$$sn(v,d)=v-\left(1+d^2\right)\frac{v^3}{3!}+\left(1+14d^2+d^4\right)\frac{v^5}{5!}-\left(1+135d^2+135d^4+d^6\right)\frac{v^7}{7!}+\ldots$$

The Finite Element Method

The domain [-1,1] × [0,٢] is divided into $M_x \times N_t$ mesh with the spatial step size $\Delta x=h=(1-(-1))/M_x=2/M_x$ and the time size $\Delta t=k=T/N_t$.,we denote $U(m\Delta x,n\Delta t)$, conveniently, by U_m^n and the nodal points (x_m,t_n) are given by $x_m=x_0+mh,\ t_n=nk$.

Remark: The following differentiation and integration results hold over the element (e):(see [8])

(i) $$\frac{\partial N_{m-1}}{\partial x}=-\frac{1}{x_m-x_{m-1}}=-\frac{1}{h}\ ,\ \frac{\partial N_m}{\partial x}=\frac{1}{x_m-x_{m-1}}=\frac{1}{h}$$

(ii) $$\int_{x_{m-1}}^{x_m} N_{m-1}^r\ N_m^t\ dx=\frac{r!\,t!\left(x_m-x_{m-1}\right)}{(r+t+1)!}$$

Where r and t are positive integers.

We use the linear piecewise approximation in the space variable and Galerkin method to obtain the semi – discrete approximation to (4)

we have

$$U^{(e)}=N_{m-1}(x)U_{m-1}(t)+N_m(x)U_m(t)$$

where

$$N_{m-1}=\frac{x_m-x}{h^{(e)}}\ ,\ N_m=\frac{x-x_{m-1}}{h^{(e)}}\ ,\ h^{(e)}=x_m-x_{m-1}$$

we also have

$$\frac{\partial U}{\partial t}=N_{m-1}\frac{dU_{m-1}}{dt}+N_m\frac{dU_m}{dt}$$

Galerkin equations in matrix form may be written as :

$$\int_{x_{m-1}}^{x_m}\left\{\begin{bmatrix} N_{m-1}N_{m-1} & N_{m-1}N_m \\ N_mN_{m-1} & N_mN_m \end{bmatrix}\begin{bmatrix} \dot{U}_{m-1} \\ \dot{U}_m \end{bmatrix}+\begin{bmatrix} N'_{m-1}N'_{m-1} & N'_{m-1}N'_m \\ N'_mN'_{m-1} & N'_mN'_m \end{bmatrix}\begin{bmatrix} U_{m-1} \\ U_m \end{bmatrix}+\right.$$

$$\begin{bmatrix} N_{m-1}\left(N_{m-1}U_{m-1}+N_mU_m\right)^3 \\ N_m\left(N_{m-1}U_{m-1}+N_mU_m\right)^3 \end{bmatrix}-(1-a)\begin{bmatrix} N_{m-1}\left(N_{m-1}U_{m-1}+N_mU_m\right)^2 \\ N_m\left(N_{m-1}U_{m-1}+N_mU_m\right)^2 \end{bmatrix}+$$

$$\left. a\begin{bmatrix} N_{m-1}N_{m-1} & N_{m-1}N_m \\ N_mN_{m-1} & N_mN_m \end{bmatrix}\begin{bmatrix} U_{m-1} \\ U_m \end{bmatrix}\right\}dx=0$$

By using the remark (i and ii) we have

$$\frac{h}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} \dot{U}_{m-1} \\ \dot{U}_m \end{bmatrix}+\frac{1}{h}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} U_{m-1} \\ U_m \end{bmatrix}+$$

$$\begin{bmatrix} N_{m-1}^4U_{m-1}^3+3N_{m-1}^3N_mU_{m-1}^2U_m+3N_{m-1}^2N_m^2U_{m-1}U_m^2+N_{m-1}N_m^3U_m^3 \\ N_{m-1}^3N_mU_{m-1}^3+3N_{m-1}^2N_m^2U_{m-1}^2U_{m-1}+3N_{m-1}N_m^3U_{m-1}U_m^2+N_m^4U_m^3 \end{bmatrix}-$$

$$(1+a)\begin{bmatrix} N_{m-1}^3U_{m-1}^2+2N_{m-1}^2N_mU_{m-1}U_m+N_{m-1}N_m^2U_m^2 \\ N_{m-1}^2N_mU_{m-1}^2+2N_{m-1}N_m^2U_{m-1}U_m+N_m^3U_m^2 \end{bmatrix}+\frac{ah}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} U_{m-1} \\ U_m \end{bmatrix}=0$$

by rearranging

$$\frac{h}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}\dot{U}_{m-1}\\ \dot{U}_m\end{bmatrix}+\frac{1}{h}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}U_{m-1}\\ U_m\end{bmatrix}+\begin{bmatrix}\frac{h}{5}U_{m-1}^3+\frac{3h}{20}U_{m-1}^2U_m+\frac{h}{10}U_{m-1}U_m^2+\frac{h}{20}U_m^3\\ \frac{h}{20}U_{m-1}^3+\frac{h}{10}U_{m-1}^2U_m+\frac{3h}{20}U_{m-1}U_m^2+\frac{h}{5}U_m^3\end{bmatrix}-$$

$$(1+a)\begin{bmatrix}\frac{h}{4}U_{m-1}^2+\frac{h}{6}U_{m-1}U_m+\frac{h}{12}U_m^2\\ \frac{h}{12}U_{m-1}^2+\frac{h}{6}U_{m-1}U_m+\frac{h}{4}U_m^2\end{bmatrix}+\frac{ah}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}U_{m-1}\\ U_m\end{bmatrix}=0$$

This implies

$$\frac{h}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}\dot{U}_{m-1}\\ \dot{U}_m\end{bmatrix}+\frac{1}{h}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}U_{m-1}\\ U_m\end{bmatrix}+\frac{h}{20}\begin{bmatrix}4U_{m-1}^3+3U_{m-1}^2U_m+2U_{m-1}U_m^2+U_m^3\\ U_{m-1}^3+2U_{m-1}^2U_m+3U_{m-1}U_m^2+4U_m^3\end{bmatrix}-$$

$$\frac{(1+a)h}{12}\begin{bmatrix}3U_{m-1}^2+2U_{m-1}U_m+U_m^2\\ U_{m-1}^2+2U_{m-1}U_m+3U_m^2\end{bmatrix}+\frac{ah}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}U_{m-1}\\ U_m\end{bmatrix}=0$$

We write the element equations for the elements $x_{m-1} \le x \le x_m$ and $x_m \le x \le x_{m+1}$ and assemble these element equations, we obtain

$$\frac{h}{6}\begin{bmatrix}2&1&0\\1&4&1\\0&1&2\end{bmatrix}\begin{bmatrix}\dot{U}_{m-1}\\\dot{U}_m\\\dot{U}_{m+1}\end{bmatrix}+\frac{1}{h}\begin{bmatrix}1&-1&0\\-1&2&-1\\0&-1&1\end{bmatrix}\begin{bmatrix}U_{m-1}\\U_m\\U_{m+1}\end{bmatrix}+$$

$$\frac{h}{20}\begin{bmatrix}4U_{m-1}^3+3U_{m-1}^2U_m+2U_{m-1}U_m^2+U_m^3\\ \left(U_{m-1}^3+U_{m+1}^3\right)+2U_m\left(U_{m-1}^2+U_{m+1}^2\right)+3U_m^2\left(U_{m-1}+U_{m+1}\right)+8U_m^3\\ U_m^2+2U_{m+1}U_m^2+3U_{m+1}^2U_m+4U_{m+1}^3\end{bmatrix}-$$

$$\frac{(1+a)h}{12}\begin{bmatrix}3U_{m-1}^2+2U_{m-1}U_m+Uu_m^2\\ \left(U_{m-1}^2+U_{m+1}^2\right)+2U_m\left(U_{m-1}+U_{m+1}\right)+6U_m^2\\ U_{m-1}^2+2U_{m-1}U_m+3U_m^2\end{bmatrix}+\frac{ah}{6}\begin{bmatrix}2&1&0\\1&4&1\\0&1&2\end{bmatrix}\begin{bmatrix}U_{m-1}\\U_m\\U_{m+1}\end{bmatrix}=0$$

Assembling the element equations and setting the row corresponding to U_m to zero . We write the difference – differential equation at the node x_m as :

$$\frac{h}{6}\left(\dot{U}_{m-1}+4\dot{U}_m+\dot{U}_{m+1}\right)+\frac{1}{h}\left(-U_{m-1}+2U_m-U_{m+1}\right)+$$
$$\frac{h}{20}\left(\left(U_{m-1}^3+U_{m+1}^3\right)+2U_m\left(U_{m-1}^2+U_{m+1}^2\right)+3U_m^2\left(U_{m-1}+U_{m+1}\right)+8U_m^3\right)-$$
$$\frac{(1+a)h}{12}\left(\left(U_{m-1}^2+U_{m+1}^2\right)+2U_m\left(U_{m-1}+U_{m+1}\right)+6U_m^2\right)+\frac{ah}{6}\left(U_{m-1}+4U_m+U_{m+1}\right)=0 \quad (6)$$

Eq.(6) can be written as

$$\frac{h}{6}\left(\dot{U}_{m-1}+4\dot{U}_m+\dot{U}_{m+1}\right)-\frac{1}{h}\left(U_{m-1}-2U_m+U_{m+1}\right)=f_m,$$

where

$$f_m=r_m+w_m+s_m,$$

and

$$w_m = -\frac{h}{20}\left(\left(U^3_{m-1} + U^3_{m+1}\right) + 2U_m\left(U^2_{m-1} + U^2_{m+1}\right) + 3U^2_m\left(U_{m-1} + U_{m+1}\right) + 8U^3_m\right),$$

$$s_m = \frac{(1+a)h}{12}\left(\left(U^2_{m-1} + U^2_{m+1}\right) + 2U_m\left(U_{m-1} + U_{m+1}\right) + 6U^2_m\right)$$

$$r_m = -\frac{ah}{6}\left(U_{m-1} + 4U_m + U_{m+1}\right),$$

for a positive integer N_t, let $\Pi_t = \{t_n\}_{n=0}^{N_t}$ be a partition of [0,T] and let $t_{n+1/2} = t_n + k/2$, for a function U defined on Π_t,

Let

$$\dot{U}_m = \frac{U^n_m - U^{n-1}_m}{k}, \quad U^{n+1/2}_m = \frac{U^n_m + U^{n-1}_m}{2}, \quad \delta^2_x = U_{m-1} - 2U_m + U_{m+1}$$

The Crank – Nicolson Galerkin scheme for the Elliptic problem (4)

$$h\left(1 + \frac{1}{6}\delta^2_x\right)\left(U^n_m - U^{n-1}_m\right) - \frac{k}{h}\delta^2_m U^{n+1/2}_m = kf\left(U^{n+1/2}_m\right), \quad n = 1,2,\ldots\ldots, N_t - 1$$

In matrix form

$$\left(A + \frac{1}{2}kB\right)U^n_m = \left(A - \frac{1}{2}kB\right)U^{n-1}_m + kf\left(U^{n+1/2}_m\right) \qquad (7)$$

Where the matrices A and B are

$$A = \frac{h}{6}\begin{bmatrix} 2 & 1 & . & . & . & . & . \\ 1 & 4 & 1 & . & . & . & . \\ . & \times & \times & \times & . & . & . \\ . & . & \times & \times & \times & . & . \\ . & . & . & \times & \times & \times & . \\ . & . & . & . & 1 & 4 & 1 \\ . & . & . & . & . & 1 & 2 \end{bmatrix}, \quad B = \frac{1}{h}\begin{bmatrix} 1 & -1 & . & . & . & . & . \\ -1 & 2 & -1 & . & . & . & . \\ . & \times & \times & \times & . & . & . \\ . & . & \times & \times & \times & . & . \\ . & . & . & \times & \times & \times & . \\ . & . & . & . & -1 & 2 & -1 \\ . & . & . & . & . & -1 & 1 \end{bmatrix}$$

where $(\times)$ and (**.**) denote a number and an empty location respectively.The matrices A and B are a tridiagonal symmetric matrices.

We observe that as n=1 since the equation (7) contain U^1 in the right side and we have to supplement it with another method for determine U^1 as initial value. This method will require a separate prescription for calculating U^1, we shall analyze here a predictor - corrector method for this purpose, using as a first approximation, the value U^1 determined by the case n=1 of equation (7) with $f\left(\frac{U_m^1+U_m^0}{2}\right)$ replaced by $f\left(U_m^0\right)$ and then as the final approximation the result of the same equation with $f\left(\frac{U_m^1+U_m^0}{2}\right)$.

i.e.

$$\left(A+\frac{1}{2}kB\right)U_m^n=\left(A-\frac{1}{2}kB\right)U_m^{n-1}+kf\left(U_m^{n-1}\right) \text{..........(predictor method)} \qquad (8)$$

$$\left(A+\frac{1}{2}kB\right)U_m^n=\left(A-\frac{1}{2}kB\right)U_m^{n-1}+kf\left(U_m^{n+1/2}\right) \text{..........(corrector method)} \qquad (9)$$

Numerical results

In this section we obtain a numerical solution of Burgers equation in the form (4), To show the efficiency of the present method for our problem in comparison with the finite difference method [7] we report absolute error which is defined by:

$$|E|=\left|u(x_i)-U_m^n(x_i,t_j)\right| \quad ,i,j=1,2,3,........N_X-1$$

In the point (x_i,t_j) where $U_m^n(x_i,t_j)$ is the solution obtained by equation (9) solved by finite element method and $u(x_i)$ is stationary solution obtained by equation (5). The code was written in matlabe power station 7 programming language.

We see that when the boundry condition b take the values 1, a, 0 the numerical solution by Crank-Nicolson finite element method and Explicit, Crank-Nicolson finite difference method are converge to the stationary solution u=1, u=a and u=0, respectively with increasing the time, and the our numerical solutions and the absolute errors are more accurate and smaller than, respectively, those of the numerical results by [7] .

The results of Numerical solution and absolute errors by (F.D.M.) and (F.E.M.) are listed in tables 1,2 and 3 with the figures(1-6) which express this results.

Table 1 : we chose a=0.3, b=1, H=0.2, $\Delta t = 0.1$

	The finite difference method [7]				The finite element method	
t	Explicit	Absolute error	Crank-Nicolson	Absolute error	Crank-Nicolson	Absolute error
0	0.4000	0.6000	0.4000	0.6000	0.4000	0.6000
0.1	0.5624	0.4376	0.5378	0.4622	0.6048	0.3952
0.2	0.6623	0.3377	0.6406	0.3594	0.7302	0.2698
0.3	0.7497	0.2503	0.7216	0.2784	0.8171	0.1829
0.4	0.8144	0.1856	0.7862	0.2138	0.8761	0.1239
0.5	0.8649	0.1351	0.8372	0.1628	0.9162	0.0838
0.6	0.9024	0.0976	0.8772	0.1228	0.9434	0.0566
0.7	0.9303	0.0697	0.9080	0.0920	0.9618	0.0382
0.8	0.9505	0.0495	0.9315	0.0685	0.9742	0.0258
0.9	0.9651	0.0349	0.9493	0.0507	0.9826	0.0174
1	0.9754	0.0246	0.9627	0.0373	0.9883	0.0117
1.1	0.9828	0.0172	0.9726	0.0274	0.9921	0.0079
1.2	0.9880	0.0120	0.9799	0.0201	0.9947	0.0053
1.3	0.9916	0.0084	0.9853	0.0147	0.9964	0.0036
1.4	0.9941	0.0059	0.9893	0.0107	0.9976	0.0024
1.5	0.9959	0.0041	0.9922	0.0078	0.9984	0.0016
1.6	0.9972	0.0028	0.9943	0.0057	0.9989	0.0011
1.7	0.9980	0.0020	0.9958	0.0042	0.9993	0.0007
1.8	0.9986	0.0014	0.9970	0.0030	0.9995	0.0005
1.9	0.9990	0.0010	0.9978	0.0022	0.9997	0.0003
2	0.9993	0.0007	0.9985	0.0015	0.9998	0.0002

Table 2 : we chose a=0.25, b=0.25, H=0.1, $\Delta t = 0.1$

	The finite difference method [7]				The finite element method	
t	Explicit	Absolute error	Crank-Nicolson	Absolute error	Crank-Nicolson	Absolute error
0	0.1000	0.1500	0.1000	0.1500	0.1000	0.1500
0.1	0.1287	0.1213	0.1274	0.1226	0.1441	0.1059
0.2	0.1573	0.0927	0.1513	0.0987	0.1779	0.0721
0.3	0.1771	0.0729	0.1706	0.0794	0.2007	0.0493
0.4	0.1933	0.0567	0.1862	0.0638	0.2163	0.0337
0.5	0.2056	0.0444	0.1987	0.0513	0.2270	0.0230
0.6	0.2152	0.0348	0.2087	0.0413	0.2343	0.0157
0.7	0.2228	0.0272	0.2167	0.0333	0.2392	0.0108
0.8	0.2287	0.0213	0.2232	0.0268	0.2426	0.0074
0.9	0.2333	0.0167	0.2284	0.0216	0.2450	0.0050
1	0.2369	0.0131	0.2326	0.0174	0.2466	0.0034
1.1	0.2397	0.0103	0.2359	0.0141	0.2476	0.0024
1.2	0.2420	0.0080	0.2387	0.0113	0.2484	0.0016
1.3	0.2437	0.0063	0.2409	0.0091	0.2489	0.0011
1.4	0.2451	0.0049	0.2426	0.0074	0.2492	0.0008
1.5	0.2461	0.0039	0.2440	0.0060	0.2495	0.0005
1.6	0.2470	0.0030	0.2452	0.0048	0.2496	0.0004
1.7	0.2476	0.0024	0.2461	0.0039	0.2498	0.0002
1.8	0.2481	0.0019	0.2469	0.0031	0.2498	0.0002
1.9	0.2485	0.0015	0.2475	0.0025	0.2499	0.0001
2	0.2488	0.0012	0.2482	0.0018	0.2499	0.0001

Table 3 : we chose a=0.45, b=0, H=0.8, $\Delta t = 0.1$

	The finite difference method [7]				The finite element method	
t	Explicit	Absolute error	Crank-Nicolson	Absolute error	Crank-Nicolson	Absolute error
0	0.8000	0.8000	0.8000	0.8000	0.8000	0.8000
0.1	0.6456	0.6456	0.6517	0.6517	0.5645	0.5645
0.2	0.4885	0.4885	0.5212	0.5212	0.3829	0.3829
0.3	0.3761	0.3761	0.4126	0.4126	0.2606	0.2606
0.4	0.2834	0.2834	0.3238	0.3238	0.1770	0.1770
0.5	0.2127	0.2127	0.2522	0.2522	0.1201	0.1201
0.6	0.1580	0.1580	0.1950	0.1950	0.0815	0.0815
0.7	0.1166	0.1166	0.1499	0.1499	0.0552	0.0552
0.8	0.0855	0.0855	0.1146	0.1146	0.0374	0.0374
0.9	0.0625	0.0625	0.0873	0.0873	0.0253	0.0253
1	0.0455	0.0455	0.0662	0.0662	0.0171	0.0171
1.1	0.0330	0.0330	0.0501	0.0501	0.0116	0.0116
1.2	0.0239	0.0239	0.0378	0.0378	0.0079	0.0079
1.3	0.0173	0.0173	0.0285	0.0285	0.0053	0.0053
1.4	0.0125	0.0125	0.0215	0.0215	0.0036	0.0036
1.5	0.0090	0.0090	0.0162	0.0162	0.0024	0.0024
1.6	0.0065	0.0065	0.0121	0.0121	0.0017	0.0017
1.7	0.0047	0.0047	0.0091	0.0091	0.0011	0.0011
1.8	0.0034	0.0034	0.0068	0.0068	0.0008	0.0008
1.9	0.0024	0.0024	0.0051	0.0051	0.0005	0.0005
2	0.0018	0.0018	0.0039	0.0039	0.0003	0.0003

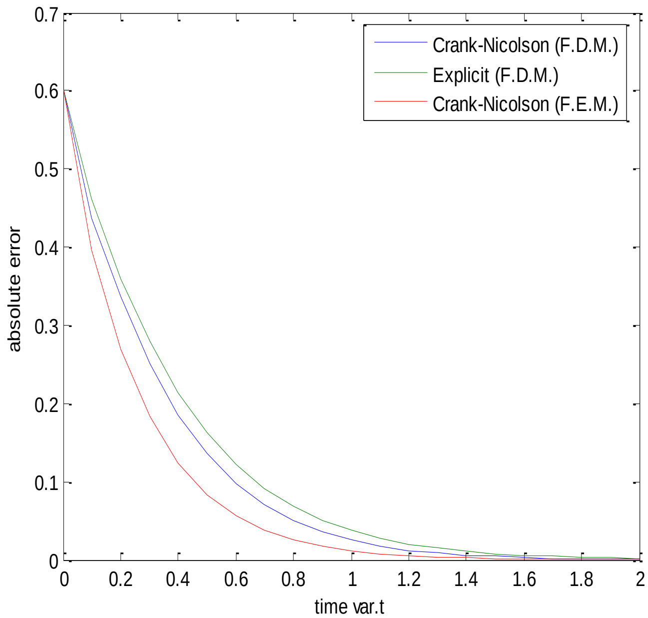

Figure 1:The absolute errors comparison of (F.E.M.)and (F.D.M.) with a=0.3, b=1, H=0.2 and $\Delta t = 0.1$

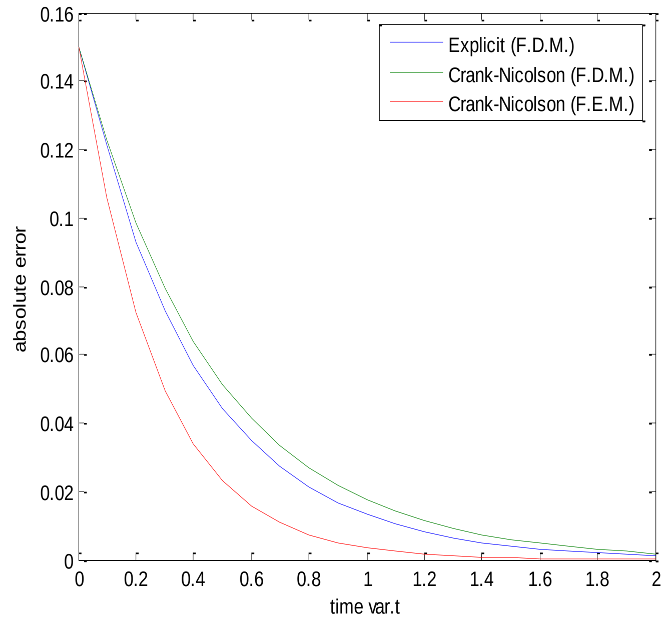

Figure 2:The absolute errors comparison of (F.E.M.)and (F.D.M.) with a=0.25, b=0.25, H=0.1 and $\Delta t = 0.1$

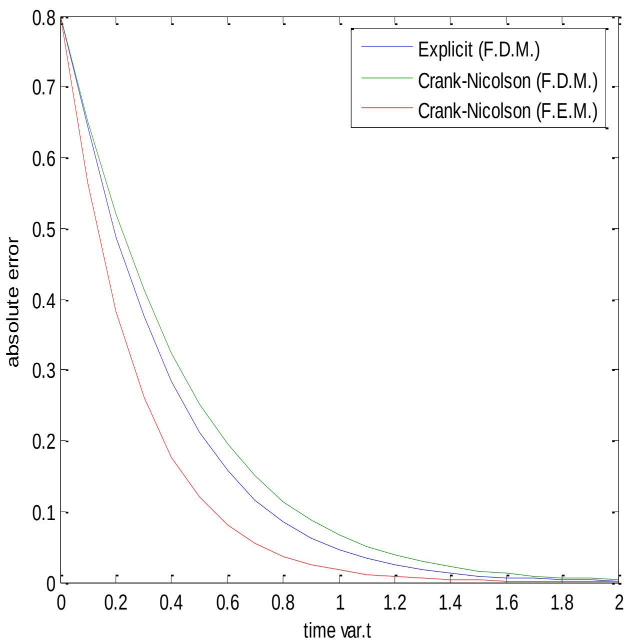

Figure 3:The absolute errors comparison of (F.E.M.)and (F.D.M.) with a=0.45, b=0, H=0.8 and $\Delta t = 0.1$

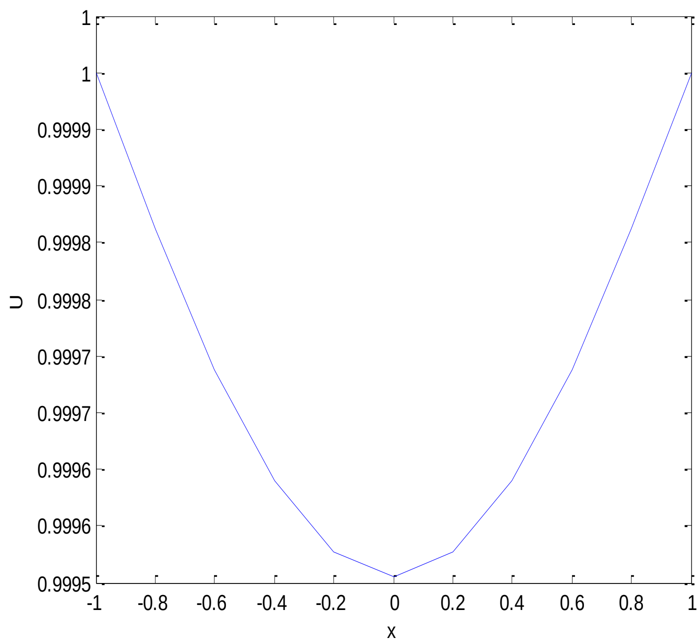

Figure 4: Numerical solution by (F.E.M.) at a=0.3, b=1, H=0.2 and $\Delta t = 0.1$

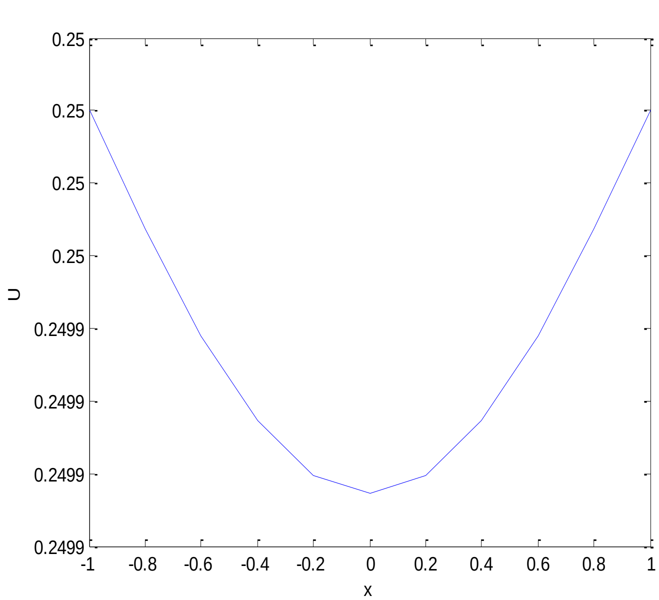

Figure 5: Numerical solution by (F.E.M.) at a=0.25, b=0.25, H=0.1 and $\Delta t = 0.1$

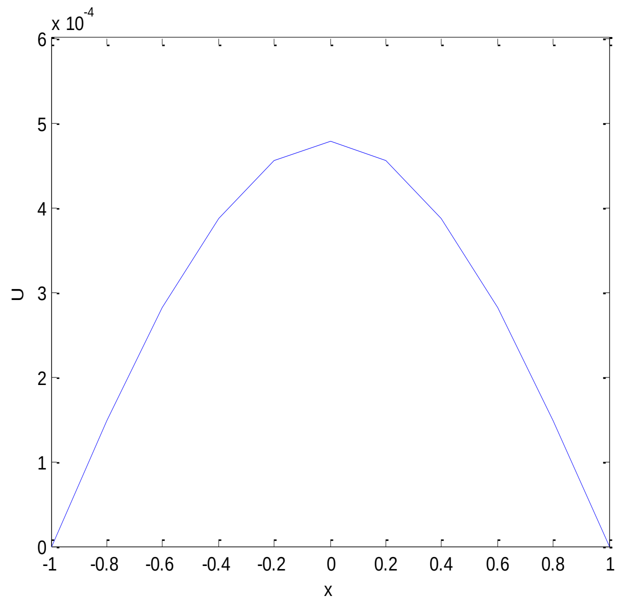

Figure ٦ Numerical solution by (F.E.M.) at a=0.45, b=0, H=0.8 and $\Delta t = 0.1$

Conclusions

We have introduced a Crank-Nicolson Galerkin finite element method for solving one dimensional non-linear Huxely equation.The method developed by predictor - corrector method to get on the better results. The numerical results and absolute errors showed that present method is more accurate than the Explicit and Crank-Nicolson finite difference method which represented by [7] .

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طريقة العناصر المحددة لحل معادلة Huxely اللا خطية

احمد جبار حسين

قسم الرياضيات – كلية التربية – جامعة ذي قار

المستخلص

تناولنا في هذا البحث حل معادلة Huxley اللاخطية بأستخدام طريقة كرانك نيكلسون (Crank Nicolson) للعناصر المحددة (Finite element) مع طريقة المخمن – المصحح (predictor - corrector method) . النتائج العددية والاخطاء المطلقة المبينة في هذا البحث هي اكثر دقة وافضل من النتائج العددية الممثلة بأستخدام الطريقة الصريحة (Explicit) وكرانك نيكلسون (Crank Nicolson) للفروقات المنتهية (Finite difference) [7] .