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0_2^+ State in ^{18}O by using Core – Polarization effects

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Abstract:

Coulomb form factors for *E0* transition in ^{18}O are discussed taking into account core-polarization effects. These effects are taken into account through the collective model of Tassie and also through a microscopic perturbation theory including excitations up to 2p1f shell. Space wave model functions defined for the orbits $1d_{5/2}$, $2s_{1/2}$ and $1d_{3/2}$ are obtained from the diagonalization of the interaction Hamilonian of Wildenthal. The calculations include the 0_2^+ state with excitation energies 3.6337MeV. The core – polarization effects which incorporate the collective model of Tassei describe the data very well for this state.

Introduction:

Comparison between calculated and measured longitudinal electron scattering form factors has long been used as stringent tests of models of transition densities.Various microscopic and macroscopic theories have been performed to study excitations in nuclei. Shell model within a restricted model space is one of the models, which succeeded in describing static properties of nuclei, when effective charges are used. In spite of the success of the 1p shell model on static properties of nuclei in this region, it fails to describe electron scattering data at high momentum transfer (Booten *et.al.*, 1994).

The ^{18}O nucleus has been the subject of extensive theoretical and experimental studies, which received much attention in last decade [Alex Brown et.al.2005]. The 18O system contains two neutrons in addition to the 16O core distributed in the sd – shell.

The dynamical features of this nucleus are then assumed to be due to the degrees of freedom of these two neutrons [Engenland 1965]. By arranging the valence neutrons within this shell and coupling them to angular momentum J=0, one can construct two 0+ states and get the monopole transitions (E0) in 18O from $(0_1^+ \rightarrow 0_2^+)$.

The Coulomb form factors have been discussed for the sd-shell nuclei using sd-shell wave functions with phenomenological effective charges [Brown et.al. 1983]. Within the context of the shell model, effective charges take the form of particle-hole excitations outside the model space defined by the active nucleons. However, the introduction of effective charges to the nucleons active in the model may bring the calculated transition strengths, which are defined at the photon point, closer to the measured values, but the non-zero momentum transfer values might deviate appreciably from the measured values.

Horsfjord,(1975) suggested that in the case of the oxygen isotopes constant polarization charges are useful up to a momentum transfer of approximately 2 fm^{-1}.

For a more practical alternative approach to use models, researchers take the collective effects into consideration. One of these models that takes the contribution of the core is the Tassie model (Tassie, 1956).

Tassie model is an attempt to obtain a model with more elasticity and modification that permits a non – uniform charge and mass density distribution. In this model, the low-lying collective modes can be described by taking the transition charge density as a radial derivative of the ground state charge density. The transition charge density of the Tassie model is essentially governed by the same parameters as the ground – state charge distribution.

The core effects can also be considered through a microscopic theory, which combine shell-model wave functions and highly excited giant resonance by using perturbation theory (Sagawa *et.al.*, 1985). Effects outside the valence model space are usually called core-polarization effects. These effects provide a more practical alternative for calculating nuclear collectivity.

Coulomb form factors of *E*4 transitions in sd-shell nuclei have been discussed in ref. (Sagawa *et.al.*, 1985), taking into account core-polarization effects. Their results gave good agreement with the experimental form factors.

The aim of the present work is to consider the 0_2^+ **state in ^{18}O with excitation energy 3.6337 MeV. The experimental data of the electron scattering for this transition in ref. Norum *et.al.*,(1982) are precise enough and span a range in momentum transfer broad enough to study nuclear structure models. The calculations are performed with the collective model, which depends on the ground state charge density (Tassie model), and with core-polarization effects, calculated in a perturbation approach including excitation up to 2p1f shell.**

Theory:

Longitudinal (Coulomb) Electron scatters form factor involving angular momentum transfer *J* and momentum transfer *q* is given by [Deforest et.al. 1966]

$$F_J^2(q) = \frac{4\pi}{2J_i+1}\frac{1}{Z^2}\left|\left\langle f \| \hat{L}_J(q) \| i \right\rangle\right|^2 F_{cm}^2 F_{fs}^2 \qquad (1)$$

where ***Ji*** is the initial spin of the nucleus. The function F_{cm} is the center of mass correction, which removes the spurious state arising from the motion of the center of mass when shell model wave functions are used and is given by [Tassie et.al. 1958]

$$F_{cm} = e^{q^2 b^2 / 4A}. \quad (2)$$

The function F_{fs} is the free nucleon form factor, which is assumed to be the same for protons and neutrons and takes the form [Willey 1963]

$$F_{fs} = e^{-0.43 q^2 / 4}. \quad (3)$$

The matrix element of the longitudinal operator between the initial (*i*) and final (*f*) states of the nucleus is

$$\langle f \| \hat{L}_J(q) \| i \rangle = \int_0^\infty dr r^2 j_J(qr) \rho_J(i,f,r) \quad (4)$$

where $\rho_J(i,f,r)$ is the nuclear transition charge density.

According to the collective modes of nuclei, the transition charge density is proportional to the ground state charge density ρ_0 as (Tassie, 1956)

$$\rho_J(i,f,r) = N r^{J-1} \frac{d\rho_0}{dr} \quad (5)$$

where *N* is a proportionality constant to be determined from the measured B(*EJ*) value. The matrix element given in equation (4) takes the form

$$\langle f \| \hat{L}_J(q) \| i \rangle = -Nq \int_0^\infty dr r^{J+1} j_{J-1}(qr) \rho_0(i,f,r) \quad (6)$$

The electric transition strength is related to the form factor at the photon point, where $q = k = E_X / \hbar c$, as

$$B(EJ) = \frac{Z^2}{4\pi} \left[\frac{(2J+1)!!}{k^J} \right]^2 F_J^2(k) \quad (7)$$

The proportionality constant N is thus determined as

$$N = -\frac{\sqrt{(2J_i+1)B(EJ)}}{2J+1} \int_0^\infty dr r^{2J} \rho_0(r) \quad (8)$$

According to the nuclear shell model, the ground state charge density is given by (Horsfjord, 1975)

$$\rho_0(r) = \sum_{n\ell j} OBDM(j, J=0) \langle j \| T_0 \| j' \rangle |R_{n\ell}(r)|^2 \quad (9)$$

where $R_{n\ell j}(r)$ is the single-particle radial wave function and $n\ell j$ sum extends over the proton orbits in the ^{16}O core. The one-body density matrix (OBDM) elements are obtained from the proton occupation numbers of the ^{16}O core, since the core neutrons as well as the valence neutrons do not contribute to charge density.

For a more practical alternative to evaluate nuclear collectivity, core-polarization calculation is performed through first-order perturbation theory.

The reduced one-body matrix element for shell model wave functions is expressed as a linear combination of the single-particle matrix elements,

$$\langle f \| \hat{L}_J(q) \| i \rangle = \sum_{j_i j_f} OBDM(i,f,J,j_i j_f) \langle j_f \| \hat{L}_J \| j_i \rangle \quad (10)$$

where the OBDM (one-body density matrix) elements are the structure factors. j_i and j_f denote the single-particle initial and final states, respectively, for the model space defined by the sd-shell orbits. The reduced single-particle matrix element of the one-body operator $\hat{L}$ may be

expressed as the sum of three terms, a sd-shell part and two core-polarization (CP) matrix elements [Brussard et.al. 1977]

$$\langle j_f \| \hat{L}_J(q) \| j_i \rangle = \langle j_f \| \hat{L}_J \| j_i \rangle + \left\langle j_f \left| \hat{L}_J \frac{Q}{E - H_0} V_{res} \right| j_i \right\rangle + \left\langle j_f \left| V_{res} \frac{Q}{E - H_0} \hat{L}_J \right| j_i \right\rangle \quad (11)$$

where the operator *Q* projects state outside the sd-shell configuration. Detailed calculations of the two core-polarization terms are given in ref. (Tassie *et.al.*, 1958). The residual interaction of the modified surface delta interaction (MSDI) is used as V_{res}. For the sd-shell, we adopt Wildenthal interaction (Wildenthal, 1984) to get the OBDM elements. The intermediate one-particle-one-hole states are taken up to 1f-2p orbits. The particle states cover the orbits of the sd and fp shells. The hole states cover all the core orbits. The single-particle wave functions are those of the harmonic oscillator (HO) potential.

The strengths of the MSDI are denoted by AT and B where T indicates the isospin (0 or 1) (Brussard *et.al.*, 1977)

The first term in equation (10) represents the contribution of the valence orbits, which gives zero contribution to the charge density for ^{18}O.

The reduced single-particle matrix element of the one-body operator is given by

$$\langle j_f \| \hat{L}_J \| j_i \rangle = \int_0^\infty dr^2 j_J(qr) \langle j_f \| Y_J \| j_i \rangle R_{n_i \ell_i}(r) R_{n_f \ell_f}(r) \quad (12)$$

Results and Discussion:

All calculations in this work incorporate the single-particle wave functions of the harmonic oscillator potential. There are different values of the oscillator size parameter b cited in the literature. The oscillator parameter was determined (Manley *et.al.*, 1990) to have the value b=1.879 $\pm$ 0.023 fm by simultaneously fitting from measurements for 15 normal-parity states in ^{18}O. The value of b deduced from the measured root mean square (rms) charge radius was found to be 1.821 fm (b_{rms}) (Brown *et.al.*, 1980). Horsfjord, (1975) used the value of b=1.7 fm to calculate the polarization charge for monopole and quadrupole excitations in ^{16}O and ^{18}O.

The form factor is very sensitive to the value of the size parameter in the absolute strength and the q dependence. So, we choose to fix this parameter for the state considered in this work and the value of b=1.879 fm (Manley *et.al.*, 1990) is adopted.

The calculated values of the OBDM elements for the ground state are given in table (1). For the transition to the $0_2^+(3.6337\ MeV)$ states from the ground state 0_1^+, their values or the valance orbits are given in table (2).

The longitudinal electron scattering form factor for this transition is shown in fig. (1), where the data are taken from ref. (Norum *et.al.*, 1982). The solid curve represents the calculations of the collective (Tassie) model derived from the shell – model ground state density.

The proportionality constant N is determined from the measured $B(C2)$.

The experimental data are well explained with the inclusion of the core – polarization effects, up to $q \cong 1.7\ fm^{-1}$, and the calculation slightly under predicts the data for higher q.

Generally the calculations reproduce the shape of the form factor and predict the diffraction minimum at its location.

The cross symbols shows the result of the core – polarization effects which incorporates first – order perturbation theory. The result underestimates the data and the position of the diffraction minimum and the entire plot are greatly shifted from the data.

However, this result reproduces the shape of the form factor. To bring the calculated strength and the q dependence to the measured values, high strength parameters must be used.

Tassie model derived from the shell – model ground state density, describe the data better than the core – polarization effects model.

Table (1): the values of the OBDM elements of the ground state of $^{18}O(0^+\ 1)$

$nlj-n'l'j'$	$1s_{1/2}-1s_{1/2}$	$1p_{3/2}-1p_{3/2}$	$1p_{1/2}-1p_{1/2}$
OBDM (T=0)	3.46410	4.89897	3.46410
OBDM (T=1)	0.0	0.0	0.0
$nlj-n'l'j'$	$1d_{5/2}-1d_{5/2}$	$2s_{1/2}-2s_{1/2}$	$1d_{3/2}-1d_{3/2}$
OBDM (T=0)	0.83362	0.202723	0.060414
OBDM (T=1)	0.68065	0.165520	0.049320

Table (2) the values of the OBDM elements of the transition $(0_1^+ \rightarrow 0_2^+)$ *3.6337 MeV* for ^{18}O.

$nlj-n'l'j'$	$1d_{5/2}-1d_{5/2}$	$2s_{1/2}-2s_{1/2}$	$1d_{3/2}-1d_{3/2}$
OBDM (T=0)	-0.29608	0.556483	-0.030869
OBDM (T=1)	-0.24175	0.45437	-0.025206

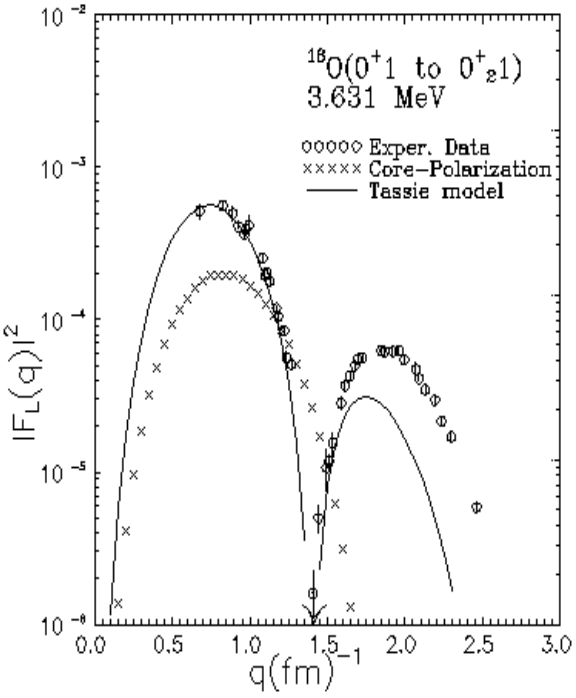

Fig.(1): Coulomb form factors for the 0_2^+ excitation In ^{18}O. The solid curve shows the Tassie model. The cross symbols shows the result with the Core – Polarization effects. The data are taken from ref. (Norum *et.al.*, 1982).

References

1- Alex Brown B. and Richter W. A., 2005. A new interaction for the sd shell., Jour. Phys., 20, 145 – 152.

2- Booten J. G. L. and Van Hees A. G.M., 1994. Magnetic electron scattering from P–shell nuclei., Nucl. Phys., A569, 510.

3- Brown B. A., Radhi R. and Wildenthal B. H.,1983. Electric quadrupole and hexadecupole nuclear excitations from the perspectives of electron scattering and modern shell – model theory., Phys. Rep., 101, 313.

4- Brown B. A., Chung W. and Wildenthal B.H., 1980. Electromagnetic multipole moments of ground states of stable odd – mass nuclei in the sd shell., Phys. Rev., C22, 774.

5- Brussard P. J. and Glaudemans P. W. M., 1977. Shell Model Application in Nuclear Spectroscopy., North Holland PublishingCompany.Amsterdam.

6- Deforest T. Jr., and Walecka J. D., 1966.Electron scattering and nuclear structure., Adv. Phys., 15, 1.

7- Engenland T., 1965. Core excitation in ^{18}O., Nucl. Phys., 72, 68.

8- Horsfjord V., 1975. Momentum dependence of polarization charges in electron excitation of nuclei in the oxygen region., Nucl. Phys., A253, 216.

9- Manley D. M., Millener D. J., Berman B. L., Bertozzi W., Buti T. N., Finn J. M., Hersman F. W., Hyde – Wright C. E., Hynes M. V., Kelly J. J., Kovash M. A., Kowalski S., Lourie R. W., Murdock B., Norum B. E., Pugh B. and Sargent C. P., 1990. Electroexcitation of rotational bands in ^{18}O., Phys. Rev., C41, 448.

10- Norum B. E., Hynes M. V., Miska H., Bertozzi W., Kelly J., Kowalski S., Rad F. N., Sargent C. P., Sasanuma T. and Turchinetz W., 1982. Inelastec electron scattering from ^{18}O., Phys. Rev., C25, 1778.

11- Sagawa H. and Brown B. A., 1985. Coulomb form factors of collective E4 transitions in s – d shell nuclei., Phys. Lett., 150B, 247.

12- Tassie L. J., 1956. A model of nuclear shape oscillations for γ – transitions and electron excitation., Aust. Jour. Phys., 9, 407.

13- Tassie L. J. and Barker F. C., 1958. Application to electron scattering of center – of – mass effects in nuclear shell model., Phys. Rev., 111, 940.

14- Willey R. S., 1963. Excitation of individual – particle states of nuclei by inelastic electron scattering., Nucl. Phys. 40, 529.

15- Wildenthal B. H., 1984. Empirical strengths of spin operators in nuclei., Prog. Part. Nucl. Phys., 11, 5.

الحالة O_2^+ لنواة الأوكسجين (١٨) باستخدام تأثيرات استقطاب القلب

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الخلاصة

حسبت عوامل التشكل الكولومية للانتقال E0 لنواة الأوكسجين (١٨) وفقاً لتأثيرات استقطاب القلب. حسبت هذه التأثيرات من خلال نموذج Tassie وكذلك تأثير استقطاب القلب من خلال نظرية الاضطراب للمرتبة الأولى التي تضمنت الانتقالات إلى القشرة 2p1f . أعطيت دوال موجة فضاء النموذج للمدارات $1d_{5/2} - 2s_{1/2} - 1d_{3/2}$ عن طريق قطر مصفوفة Hammilton مستخدمين تفاعل Wildenthal . الحسابات تضمنت الحالة 2_1^+ ، بطاقة تهيج مقدارها 3.6337 MeV . تأثير استقطاب القلب من خلال نموذج Tassie وصف الحالات بصورة جيدة.