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Four-Wave Mixing Conversion in QD SOA: reservoir effects

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Abstract:

In this paper, it is present a model to simulate Four-wave mixing in quantum dot semiconductor optical amplifier, taking into account the influence of carrier heating at reservoir. The numerical calculations show that; the carrier heating relaxation at reservoir demonstrates a significant impact on excited state occupation probability, and opposite occurs with ground state. Also, the conversion efficiency is shown a good match with experiment data.

Keyword: semiconductor optical amplifier, quantum dot structure, carrier heating and four-wave mixing.

1. Introduction:

Semiconductor amplifiers are known for optical nonlinear effects, such as the Four-wave mixing (FWM). FWM in SOA's has been used as a process that performing wavelength conversion (WC) due to its good conversion efficiency and high speed response for wavelength division multiplexing networks [T. W. Berg *et al*.,2004]. It is a process by which optical signals at different (but closely spaced) wavelengths mix to produce new signals at other wavelengths, FWM has three different physical mechanisms contributing toward its conversion. They are carrier density pulsation (CDP), spectral hole burning (SHB) and carrier heating (CH). CDP results from the beating between the pump and probe. Near the signal wavelength, carrier depletion is experienced by all the carriers in the active region, thus, the gain is reduced over a wide spectral range which shifts the steady-state global quasi-Fermi levels (that results from the contribution of both ground state (GS), excited state (ES), and wetting layer (WL)) [J. Kim *et al*.,2008]. The 2nd mechanism, SHB, is originated from ES to GS, it has an ultrafast recovery time [J. Kim *et al*., 2010], and occurs as the strong pump preferentially depletes resonant carriers while leaving carriers in other energy states unaffected, creating a spectral hole around the signal wavelength. To return to quasi-equilibrium, carriers relax down into the depleted states via carrier-carrier scattering. This process is very fast with relaxation time of about tens of femtosecond. The last FWM mechanism is CH, in which the temperature of the carriers is raised above that of the lattice and must cool down through carrier-phonon interactions. In quantum dots (QDs), the carrier density is lower than bulk and quantum well (QW) devices, then CH is reduced. Experimental measurements shows that CH is reduced in QDs [J. Kim *et al*., 2008 and D. Nielsen *et al*., 2010].

Although QDs shows promising properties when used in the active regions of semiconductor devices, some of its optical properties still limited down to that of QW devices. For example, their modulation bandwidth is limited to 10-12GHz within 1.3-1.6 μm operating wavelength. It is found that the carrier capture from optical confinement layer (barrier) can strongly limits its characteristics [C. Wang *et al*., 2012].

In this work, a numerical model is derived to study FWM efficiency in three-level rate equation system (including WL, ES and GS), taking to in account the effect of carrier heating in reservoir that was not studied earlier.

2. The Rate Equations:

Despite of a large number of describing. The dynamic of carrier in QD SOA, the effect of CH in this process dons not taken.

The dynamic of electron in reservoir, taking in to account CH, can be written

$$\frac{dN_w}{dt} = \frac{I}{eV} - \frac{N_w(1-h)}{\tau_{w2}} + \frac{N_w h}{\tau_{2w}} - \frac{N_w}{\tau_{wr}} - \frac{N_w}{\tau_{CH}} \qquad (1)$$

where N_w is the carrier density, I is the electrical current, e is the electron charge, V is the volume of active region, h is the occupation probability of excited state (ES), τ_{w2}, τ_{2w}, τ_{wr}, τ_{CH} are the time relaxation of electron QD structure from reservoir to ES level, the time relaxation from ES level to reservoir, the spontaneous recombination lifetime in WL and the carrier heating relaxation time respectively.

The rate of occupation probability of ground state (GS) and ES are expressed by the following equations [Ahmed *et al*., 2013]

$$\frac{dh}{dt} = \frac{N_w L_w(1-h)}{NQ\,\tau_{w2}} + \frac{f(1-h)}{\tau_{12}} - \frac{N_w L_w h}{NQ\,\tau_{2w}} - \frac{h(1-f)}{\tau_{21}} \qquad (2)$$

$$\frac{df}{dt} = \frac{h(1-f)}{\tau_{21}} - \frac{f(1-h)}{\tau_{12}} - \frac{f^2}{\tau_{1r}} - a(2f-1)S \qquad (3)$$

L_w is the thickness of active region, τ_{1r} is the spontaneous radiative lifetime in the QDs. *NQ* is the surface density of QDs, τ_{21} is the electron relaxation time from the ES to the GS , τ_{12} is the electron escape time from the GS to the ES, S is the photon density and $a = (g / NQ)$ is the gain factor.

3. Theory of FWM in QD SOA:

Assume that a strong optical signal (pump) with angular frequency ω_0 and a week optical signal (probe) with angular frequency ω_1 are injected to in the input facet of the optical amplifier as illustrated in figure (1).

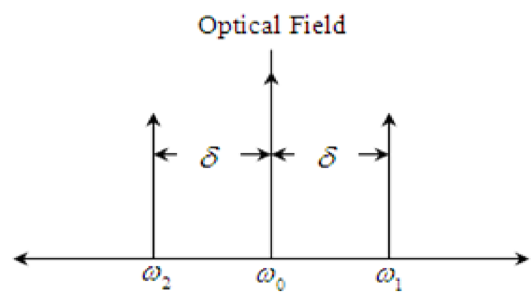

Figure (1): Optical field of pump, probe and conjugate versus detuning [A. Uskov et al., 1994].

Due to high nonlinearity of semiconductor, the conjugated fields are generated at combination frequency $\omega_n = \omega_0 + n.\delta$, $n = \pm 1, \pm 2, ...$, where δ is the detuning frequency. Because the strong pump signal comparing with other fields, the fields after conjugate signal ($\omega_2 = \omega_0 - \delta$) can be neglected [D. Nielsen *et al*., 2010].

Fellow the method of [O. Qasaimeh,2004] and using the concept of CH at reservoir, we derived the wave-mixing in QD SOA. Assume that, the photon density, occupation probability and carrier density can be represented as [O. Qasaimeh, 2004 and Ahmed H. Flayyih, 2012],

$$f_{GS} = \bar{f}_{GS} + \Delta f_{GS}\, e^{-i\delta t} + c.c \qquad (4)$$

$$f_{ES} = \bar{f}_{ES} + \Delta f_{ES}\, e^{-i\delta t} + c.c \qquad (5)$$

$$N_{wL} = \bar{N}_{wL} + \Delta N_{wL} e^{-i\delta t} + c.c \qquad (6)$$

$$S = \bar{S} + \Delta S e^{-i\delta t} + c.c \qquad (7)$$

where, $\bar{f}_{GS}$, $\bar{f}_{ES}$, $\bar{N}_{wL}$ and $\bar{S}$ (Δf_{GS} , Δf_{ES} and ΔN_{wL}) are the occupation of GS, ES , carrier density and photon density at steady state (small signal), respectively.. Substituting eqs. (4, 5, 6 , 7) in eqs.(1,2 , 3), one obtains,

$$\Delta f_{GS} = \frac{-\left(a\left(2\bar{f}_{GS}-1\right)\Delta S\right)}{\left[-i\delta + \frac{\bar{f}_{ES}}{\tau_{21}} + \frac{(1-\bar{f}_{ES})}{\tau_{12}} + 2a\bar{S} - \frac{\Delta f_{ES}}{\Delta f_{GS}}\left(\frac{(1-\bar{f}_{GS})}{\tau_{21}} + \frac{\bar{f}_{GS}}{\tau_{12}}\right)\right]} \quad (8)$$

where

$$\frac{\Delta f_{ES}}{\Delta f_{GS}} = \frac{\left(\frac{(1-\bar{f}_{ES})}{\tau_{12}} + \frac{\bar{f}_{ES}}{\tau_{21}}\right)}{\left[\begin{array}{l}-i\delta + + \frac{\bar{f}_{GS}}{\tau_{12}} + \frac{(1-\bar{f}_{ES})}{\tau_{21}} + \frac{\bar{N}_{WL}}{D}\left(\frac{1}{\tau_{w2}} + \frac{1}{\tau_{2w}}\right)\times \\ \left\{1-\left(\frac{(1-\bar{f}_{ES})}{\tau_{w2}} - \frac{\bar{f}_{ES}}{\tau_{2w}}\right)\left[-i\delta + \frac{(1-\bar{f}_{ES})}{\tau_{w2}} - \frac{\bar{f}_{ES}}{\tau_{2w}} + \frac{1}{\tau_{wr}} + \frac{1}{\tau_{CH}}\right]^{-1}\right\}\end{array}\right]} \quad (9)$$

The influence of CH represented by τ_{CH} can be noted in Eq.(9), which is not taking into account in [O. Qasaimeh,2004]. To calculate the effective pulse propagation of wave-mixing (pump, probe and conjugate), it assume an electric field [A. Uskov *et al*.,1994],

$$E(t) = E_0 e^{-i\omega_0 t} + E_1 e^{-i\omega_1 t} + E_2 e^{-i\omega_2 t} + c.c \quad (10)$$

E_0, E_1 and E_2 are the time independent amplitude of the pump, probe and the conjugate formed through nonlinear mixing. The variation of the total electric intensity along the amplifier length is given as [O. Qasaimeh, 2004],

$$\frac{\partial E}{\partial z} = \frac{E}{2}\left[-\alpha_\ell + (1-j\alpha_H) g_0 \left(2 f_{GS} - 1\right)\right] \quad (11)$$

where α_ℓ is the waveguide loss, α_H is the linewidth enhanced factor, g_0 is the model gain and $j = \sqrt{-1}$. Substituting eq. (10) in to eq.(11). For the small signal response of eq.(10), the terms which are resonant with $e^{-i\delta\tau}$, are

$$\frac{dE_{1,2}}{dz} = \frac{E_{1,2}}{2}\left[\begin{array}{l}-\alpha_\ell + (1-j\alpha_H) g_0 \left[2\bar{f}_{GS}-1\right] - a(2\bar{f}_{GS}-1) \\ \left[-i\delta + \frac{\bar{f}_{ES}}{\tau_{21}} + \frac{(1-\bar{f}_{ES})}{\tau_{12}} + 2a\bar{S} - \frac{\Delta f_{ES}}{\Delta f_{GS}}\left(\frac{(1-\bar{f}_{GS})}{\tau_{21}} + \frac{\bar{f}_{GS}}{\tau_{12}}\right)\right]|E_0|^2\end{array}\right]$$
$$-\frac{1}{2}\left[-i\delta + \frac{\bar{f}_{ES}}{\tau_{21}} + \frac{(1-\bar{f}_{ES})}{\tau_{12}} + 2a\bar{S} - \frac{\Delta f_{ES}}{\Delta f_{GS}}\left(\frac{(1-\bar{f}_{GS})}{\tau_{21}} + \frac{\bar{f}_{GS}}{\tau_{12}}\right)\right] a(2\bar{f}_{GS}-1)\left(E_{2,1}^{*} E_0^{2}\right) g_0 \quad (12)$$

In eq.(12), the influence of CH is present in the term $\frac{\Delta f_{ES}}{\Delta f_{GS}}$, this is not dealt in previous studies [D. Nielsen et al., 2010, A. Uskov et al.,1994 and O. Qasaimeh,2004]. The ratio of conjugate field intensity $E_2(z=L)$ to the input probe field intensity $E_1(z=0)$ is defines the conversion efficiency of FWM [O. Qasaimeh, 2004].

4. Results and discussion:

The numerical simulation is achieved for InAs QD grown on InGaAs which is lattice match with GaAs. The semiconductor device has 5mm in high, 20 μm of with and 10 nm layer thickness [D. Nielsen *et al.*, 2010]. The values of parameters at room temperature are listed in Tab (1). To perform the numerical calculations, a strong pump pulse with a Gaussian shape and full width at half maximum of (1 ps) has been sent through QD SOA.

Tab.(1): parameters for numerical calculations used in the simulation [D. Nielsen et al., 2010, A. Uskov et al., 1994 and O. Qasaimeh,2004].

Name	Value	Unit	Name	Value	Unit
τ_{12}	1.2	Ps	a	$5.6*10^{13}$	m^2
τ_{21}	0.16	Ps	α_H	1	
τ_{1r}	0.2	ns	α_ℓ	320	M
τ_{w2}	3	ps	I	150	mA
τ_{2w}	1	ns	g_{max}	1490	m^{-1}
τ_{wr}	0.4	ns	ΔE	80	meV
τ_{CH}	2.5	ps	NQ	$5*10^{14}$	m^{-2}

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FWM in SOA depends on three principal contributions, which are carrier density pulsation, spectral hole burning and carrier heating. In spite of the importance of CH contribution in wave-mixing, it did not take enough attention in the most of studies [D. Nielsen *et al.*, 2010 , O. Qasaimeh, 2004]. In current study, we introduce a theoretical model to simulate FWM process in QD SOA taking in to account CH contribution in reservoir.

Figure (1) shows the influence of CH on occupation probabilities of GS is slightly changed compared to ES occupation probability, the reason behind this behavior is the fast relaxation from ES to GS $\left(\tau_{21} = .16\, ps\right)$ that compensates the consumption of carriers due to recombination process and carrier escape form GS to ES. The reduction of ES occupation probability can be interpreted by reduction of carriers between reservoir (WL) and ES, as is illustrated in Figure (2).

Also, the trend in Figure (1) reflects behavior of the recovery time; it increases straight forward with CH effect. FWM conversion efficiency of QD SOA has been investigated in current study, taking into accounts the influence of carrier heating in reservoir. Figure (3) shows the effect of carrier heating relaxation time on the FWM conversion efficiency, with increasing carrier heating relaxation time, the conversion efficiency of FWM increases. Simply, the carriers at reservoir reduce (as in Figure (2)), and therefore the conversion expects to be less. Figure (4) shows a good agreement with experiment data (red circles) [T. Akiyama *et al.*, 2002] when effect of CH was taken in to account.

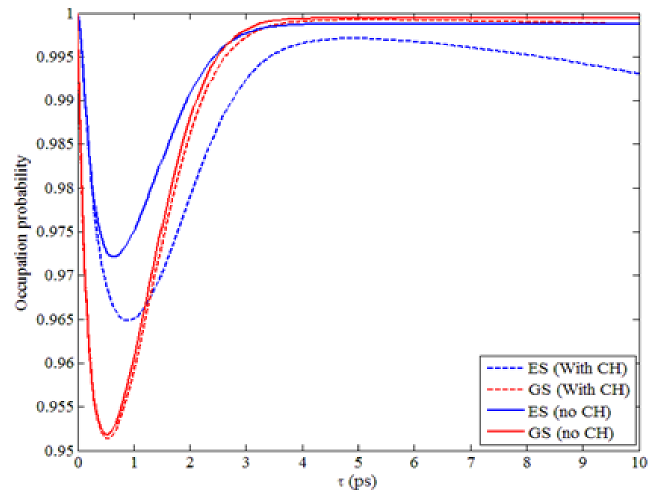

Figure (1): shows the series of occupation probability.

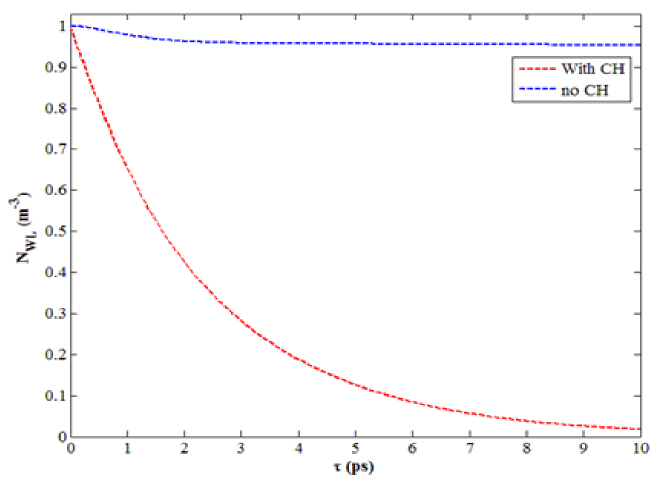

Figure (2): the time series of carrier density.

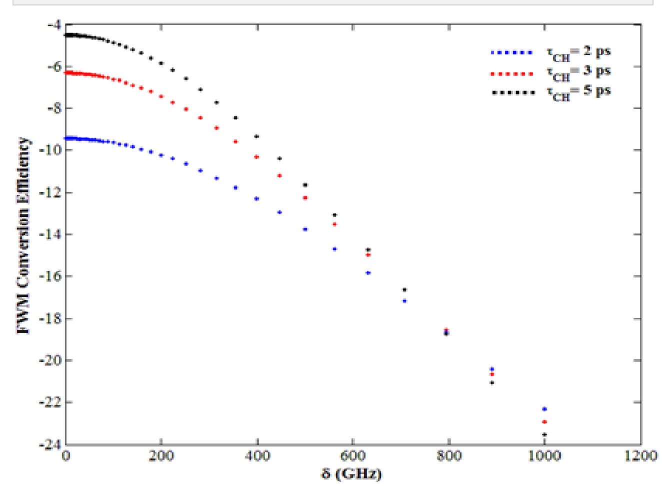

Figure (3): illustrates the effect of CH on conversion efficiency of FWM.

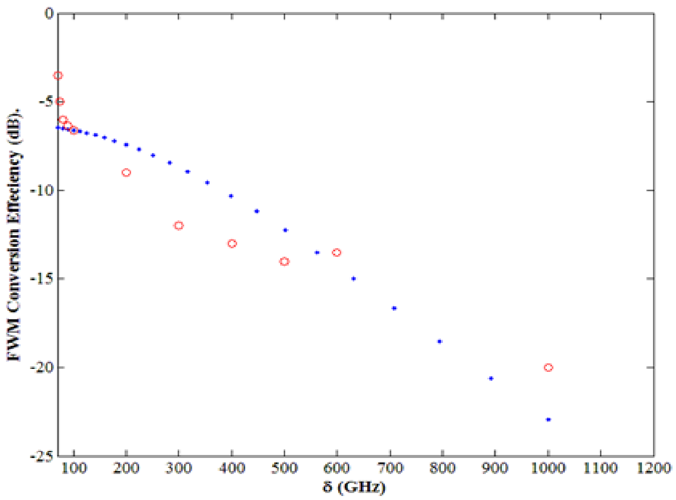

Figure (4): shows the theoretical and experimental matching.

5. Conclusions:

We are conclude that ,the effect of CH in reservoir causes a reduction in the carrier (electrons) in WL and ES level in QD structure. This leads to reduce the FWM efficiency in these devices.

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