J.Thi-Qar Sci. No. (2) Vol.1 Aug./2008

ISSN 1991- 8690 الترقيم الدولي ١٩٩١ - ٨٦٩٠
website: http://jsci.utq.edu.iq Email: utjsci@utq.edu.iq

Some types of fuzzy ideals in semigroups

Rabee Hadi Jari Akram Barazan Attar

Department of Mathematics - College of Education

University of Thi-Qar.

Abstract

In this paper ,we study the notion of fuzzy ideal in semigroups and give some properties about it and we reviewed some types of ideals such as (regular,semiprime,(1,2)- ideal,(2,2)-ideal and gives some relationships between them.

1. Introduction:

As a continuation to the study of fuzzy sets which initiated in[1]and which have been studied by Goguen in [2] and several researchers explored on the generalization of the notion of fuzzy set.The concept of an intuitionistic fuzzy set was introduced byAtanssov K.Y.[3],as a generalization of the notion of fuzzy set.Fuzzy ideals and fuzzy bi-ideal,(1,2)-ideals in semigroup was introduced by S.Lajos and N.Kuroki [4,5].Our interest in this paper is to study some of their important properties.

2. Preliminaries:

In this section ,we shall give the concepts of fuzzy sets and basic definitions with some related properties which will be used in the next sections.

Definition 2.1[1]:

A fuzzy set in a set *M* is a mapping *X* from a nonempty set *M* into [0, 1].

Definition 2.2 [1] :

Let *A* and *B* be two fuzzy sets in *M*.Then:

1.$A \subseteq B$ if and only if $A(x) \leq B(x)$

2.$(A \cap B)(x) = min \{A(x), B(x)\}$, for all $x \in M$.

Definition 2.3[6]:

A fuzzy set *X* in a ring *R* is called a fuzzy ideal of *R* if for each $x,y \in R$.

1- $X(x-y) \geq min \{X(x), X(y)\}$.

2- $X(xy) \geq max \{X(x), X(y)\}$.

3. Intuitionistic Q- fuzzy ideals

Definition 3.1[7]:

An intuitionistic Q-fuzzy set A is an object having the form $A = \{(x, \mu_A(x,q), \gamma_A(x,q)) : x \in X, q \in Q\}$ where the function $\mu_A : X \times Q \rightarrow [0,1]$ and $\gamma_A : X \times Q \rightarrow [0,1]$ denoted by the degree of membership(namely $\mu_A(x,q)$) and the degree of nonmembership $\gamma_A(x,q)$ of each element $(x,q) \in X \times Q$ to the set A, respectively, and $0 \leq \mu_A(x,q) + \gamma_A(x,q) \leq 1$ for all $x \in X$ and $q \in Q$.

For the sake of simplicity, we shall use the symbol $A=(\mu_A,\gamma_A)$ for the IQFS $A=\{(x,\mu_A(x,q),\gamma_A(x,q)):x\in x,q\in Q\}$.

Definition 3.2 [7] :

An IQFS $A=(\mu_A,\gamma_A)$ in S is called an intuitionistic Q-fuzzy subsemigroup of S if

1. $\mu_A(xy,q)\geq \min\{\mu_A(x,q),\mu_A(y,q)\}$.

2. $\gamma_A(xy,q)\leq \max\{\gamma_A(x,q),\gamma_A(y,q)\}$,for all $x,y\in S$ and $q\in Q$.

Definition 3.3 [7]:

An IQFS $A=(\mu_A,\gamma_A)$ in S is called an intuitionistic Q-fuzzy left ideal of S if

1. $\mu_A(xy,q)\geq \mu_A(y,q)$,

2. $\gamma_A(xy,q)\leq \gamma_A(y,q)$, *for* all $x, y\in S$ and $q\in Q$.

Definition 3.4 [7]:

An intuitionistic Q-fuzzy intuitionistic subsemigroup $A=(\mu_A,\gamma_A)$ of S is called an intuitionistic Q-fuzzy bi-ideal of S if

1. $\mu_A(xwy,q)\geq \min\{\mu_A(x,q),\mu_A(y,q)\}$,

2. . $\gamma_A(xwy,q)\leq \max\{\gamma_A(x,q),\gamma_A(y,q)\}$,for all $x,y,w\in S$ and $q\in Q$.

Proposition 3.5:

If A_i is an intuitionistic Q-fuzzy bi-ideal of S $\forall\, i\in\wedge$,then $\cap A_i$ is an intuitionistic Q-fuzzy bi-ideal ,where $\cap A_i=(\wedge\mu_{Ai},\vee\gamma_{Ai})$ and $\wedge\,\mu_{Ai}(x)=\inf\{\mu_{Ai}(x)/i\in\wedge, x\in S\}$, $\vee\,\gamma_{A_i}(x)=\sup\{\gamma_{A_i}(x)/i\in\wedge,x\in S\}$.

Proof:

To prove $\cap A_i$ is an intuitionistic Q-fuzzy subsemigroup

$\wedge\,\mu_{A_i}(xy,q)\geq\wedge\{\min\{\mu_{A_i}(x,q),\mu_{A_i}(y,q)\}\}$

$=\min\{\min\{\mu_{A_i}(x,q),\mu_{A_i}(y,q)\}\}=\min\{\min\{\mu_{A_i}(x,q),\min\,\mu_{A_i}(y,q)\}\}=$

$\min\{\wedge\{\mu_{A_i}(x,q),\wedge\mu_{A_i}(y,q)\}\}$

and

$\vee\,\gamma_{A_i}(xy,q)\leq\vee\{\max\{\gamma_{A_i}(x,q),\gamma_{Ai}(y,q)\}\}=\max\{\max\{\gamma_{A_i}(x,q),\gamma_{A_i}(y,q)\}\}=$

$\{\max\{\gamma_{A_i}(x,q),\max\,\gamma_{A_i}(y,q)\}\}=\max\{\vee\{\gamma_{A_i}(x,q),\vee\gamma_{A_i}(y,q)\}\}$.

Thus, $\cap A_i$ is an intuitionistic Q-fuzzy intuitionistic subsemigroup of S.

To prove $\cap A_i$ is Q-fuzzy bi-ideal of S.

Let $x, y, a \in S, then\ \mu_{A_i}(xay, q) \geq \wedge\{\min\{\mu_{A_i}(x, q), \mu_{A_i}(y, q)\}\} =$

$\min\{\min\{\mu_{A_i}(x, q), \mu_{A_i}(y, q)\}\} = \min\{\min\{\mu_{A_i}(x, q), \min \mu_{A_i}(y, q)\}\} =$

$\min\{\wedge\{\mu_{A_i}(x, q), \wedge\mu_{A_i}(y, q)\}\}$

and

$\vee\gamma_{A_i}(xay, q) \leq \vee\{\max\{\gamma_{A_i}(x, q), \gamma_{A_i}(y, q)\}\} = \max\{\max\{\gamma_{A_i}(x, q), \gamma_{A_i}(y, q)\}\} =$

$\max\{\max\{\gamma_{A_i}(x, q), \max \gamma_{A_i}(y, q)\}\} = \max\{\vee\gamma_{A_i}(x, q), \vee\gamma_{A_i}(y, q)\}.$

Hence, $\cap A_i$ is intuitionistic Q-fuzzy bi-ideal of S.

Definition 3.6 [8]:

An intuitionistic Q-fuzzy intuitionistic subsemigroup $A = (\mu_A, \gamma_A)$ of S is called an intuitionistic fuzzy (1,2)-ideal of S if

1. $\mu_A(xw(yz), q) \geq \min\{\mu_A(x, q), \mu_A(y, q), \mu_A(z, q)\}$,
2. $\gamma_A(xw(yz), q) \leq \max\{\gamma_A(x, q), \gamma_A(y, q), \gamma_A(z, q)\}$ for all w,x,y,z $\in$ S and q $\in$ Q.

Proposition 3.7:

Every intuitionistic Q-fuzzy bi-ideal is an intuitionistic Q-fuzzy (1,2)-ideal.

Proof:

Let $A = (\mu_A, \gamma_A)$ be an intuitionistic Q-fuzzy intuitionistic bi-ideal of S and let $w, x, y, z \in S$ and $q \in Q$,then

$\mu_A(xw(yz), q) = \mu_A(x(wy)z, q) \geq \min\{\mu_A((xwy), q), \mu_A(z, q)\} \geq$

$\min\{\min\{\mu_A(x, q), \mu_A(y, q), \mu_A(z, q)\}\} = \min\{\{\mu_A(x, q), \mu_A(y, q), \mu_A(z, q)\}\}$

and

$\gamma_A(xw(yz), q) = \gamma_A(xwy)z, q) \leq \max\{\gamma_A((xwy), q), \gamma_A(z, q)\} =$

$\max\{\gamma_A(x, q), \gamma_A(y, q), \gamma_A(z, q)\}.$

Thus, $A = (\mu_A, \gamma_A)$ is an intuitionistic Q-fuzzy (1,2)-ideal of S.

Recall that a semigroup S is said to be regular if, for each $x \in S$,there exists $y \in S$ such that x=xyx,[9].

Proposition 3.8:

If S is a regular semigroup,then every intuitionistic Q-fuzzy (1,2)-ideal of S is an intuitionistic Q-fuzzy bi-ideal.

Proof:

Suppose that S is a regular semigroup and let $A=(\mu_A,\gamma_A)$ be an intuitionistic Q-fuzzy (1,2)-ideal of S.

Let $w,x,y\in S$,since S regular, then $xw\in(xSx)S\subseteq xS$,it follows xw=xsx for some $s\in S$,consequently

$\mu_A(xwy,q)=\mu_A((xsx)y,q)=\mu_A(xs(xy),q)\geq\min\{\mu_A(x,q),\mu_A(x,q),\mu_A(y,q)\}=$
$\min\{\mu_A(x,q),\mu_A(y,q)\}$

and

$\gamma_A((xwy),q)=\gamma_A((xsx)y,q)=\gamma_A(xs(xy),q)\leq\max\{\gamma_A(x,q),\gamma_A(x,q),\gamma_A(y,q)\}=$
$\max\{\gamma_A(x,q),\gamma_A(y,q)\}$

Hence $A=(\mu_A,\gamma_A)$ is intuitionistic Q-fuzzy bi-ideal of S.

Recall that, A semigroup S is said to be (2,2)-regular if $x\in x^2Sx^2$ for all $x\in S$,[9,10].

Proposition 3.9:

Let $A=(\mu_A,\gamma_A)$ be an intuitionistic Q-fuzzy bi-ideal of S.If S is a (2,2)-regular, then $A(a,q)=A(a^2,q)$ for all $a\in S$.

Proof:

Let $a\in S$,since S is (2,2)-regular, then $a\in a^2Sa^2$,implies that $a=a^2xa^2$ for some $x\in S$.

Now, $\mu_A(a,q)=\mu_A(a^2xa^2,q)\geq\min\{\mu_A(a^2,q),\mu_A(a^2,q)\}=\mu_A(a^2,q)$
$\geq\min\{\mu_A(a,q),\mu_A(a,q)\}=\mu_A(a,q)$

also,

$\gamma_A(a,q)=\gamma_A(a^2xa^2,q)\leq\max\{\gamma_A(a^2,q),\gamma_A(a^2,q)\}=\gamma_A(a^2,q)\leq\max\{\gamma_A(a,q),\gamma_A(a,q)\}=$
$\gamma_A(a,q)$.

Hence , $\mu_A(a,q)=\mu_A(a^2,q)$ and $\gamma_A(a,q)=\gamma_A(a^2,q)$;that is $A(a,q)=A(a^2,q)$.

Recall that, a semigroup S is called intra-regular if, for each element a of S,there exists elements x and y in S such that $a = xa^2y$,[9].

Proposition 3.10:

Let $A = (\mu_A, \gamma_A)$ be an intuitionistic Q-fuzzy ideal of S.If S is an intra-regular, then $A(a,q) = A(a^2,q)$ for all $a \in S$ and $q \in Q$.

Proof:

L et $a \in S$,since S is intra-regular, there exist x and y $\in S$ such that $a = xa^2y$.

Now, $\mu_A(a,q) = \mu_A(xa^2y,q) \geq \mu_A(xa^2,q) \geq \mu_A(a^2,q) \geq \{\mu_A(a,q), \mu_A(a,q)\} = \mu_A(a,q)$.

Also, $\gamma_A(a,q) = \gamma_A(xa^2y,q) = \gamma_A(xa^2,q) \leq \gamma_A(a^2,q) \leq \max\{\gamma_A(a,q), \gamma_A(a,q)\} = \gamma_A(a,q)$.

Since $\mu_A(a,q) = \mu_A(a^2,q)$ and $\gamma_A(a,q) = \gamma_A(a^2,q)$,then $A(a,q) = A(a^2,q)$.

Corollary3.11 ;

Let $A = (\mu_A, \gamma_A)$ be an intuitionistic Q-fuzzy ideal of S.If S is intra-regular, then $A(ab,q) = A(ba,q)$ for all $a,b \in S$.

Proof:

Let $a,b \in S$. Then by Proposition 3.10,

$$\mu_A(ab,q) = \mu_A((ab)^2,q) \geq \mu_A((a(ba)b,q) \geq \mu_A(ba,q) = \mu_A((ba)^2,q)$$
$$\geq \mu_A(b(ab)a,q) \geq \mu_A(ab,q).$$

and

$$\gamma_A(ab,q) = \gamma_A((ab)^2,q) = \gamma_A(a(ba)b,q) \leq \gamma_A(ba,q) = \gamma_A((ba)^2,q) =$$
$$\gamma_A(b(ab)a,q) \leq \gamma_A(ab).$$

Hence $\mu_A(ab,q) = \mu_A(ba,q)$ and $\gamma_A(ab,q) = \gamma_A(ba,q)$,so $A(ab,q) = A(ba,q)$.

4. Intuitionistic fuzzy ideals

Definition 4.1[10]:

An intuitionistic fuzzy set (brifly,IFS) A in a non-empty set X is an object having the form $A = \{(x, \mu_a(x),, \gamma_A(x)) : x \in X\}$ where the function $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ denoted by the degree of membership and the degree of non membership,respectively,and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for all $x \in X$ .An intuitionistic fuzzy set $A = \{(x, \mu_A(x), \gamma_A(x)) : x \in X\}$ in X can be identified to an order pair (μ_A, γ_A) in $I^X \times I^X$ .For the sake of simplicity, denoted by the symbol $A = (\mu_A, \gamma_A)$ for the IFS $A = \{(x, \mu_A(x),, \gamma_A(x)) : x \in X\}$.

Definition 4.2 [9]:

An IFS $A = (\mu_A, \gamma_A)$ in S is called an intuitionistic fuzzy subsemigroup of S if

1. $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$.
2. $\gamma_A(xy) \leq \max\{\gamma_A(x), \gamma_A(y)\}$,for all $x, y \in S$.

Definition 4.3 [9]:

An IFS $A = (\mu_A, \gamma_A)$ in S is called an intuitionistic fuzzy left ideal of S if

1. $\mu_A(xy) \geq \mu_A(y)$,
2. $\gamma_A(xy) \leq \gamma_A(y)$, *for* all x, y $\in$ S.

Definition 4.4 [3]:

An IFS $A = (\mu_A, \gamma_A)$ in S is called intuitionistic fuzzy semiprime if

1. $\mu_A(x) \geq \mu_A(x^2)$
2. $\gamma_A(x) \leq \gamma_A(x^2)$,for all $x \in S$.

Proposition 4.5 :

For any intuitionistic fuzzy subsemigroup $A = (\mu_A, \gamma_A)$ of S,if $A = (\mu_A, \gamma_A)$ is intuitionistic fuzzy semiprime,then $A(a) = A(a^2)$.

Proof:

Let $a \in S$ .Then, $\mu_A(a) \geq \mu_A(a^2) = \min\{\mu_A(a), \mu_A(a)\} = \mu_A(a)$.

So, $\mu_A(a) = \mu_A(a^2)$

And $\gamma_a(a) \leq \gamma_A(a^2) = \max\{\gamma_A(a), \gamma_A(a)\} = \gamma_A(a)$.

Hence, $\gamma_A(a) = \gamma_A(a^2)$,it follows that $A(a) = A(a^2)$.

Recall that a semigroup is called left regular if for each element a of S,there exists an element x in S such that $a = xa^2$,[9].

Proposition 4.6:

Let S be left regular,then every intuitionistic fuzzy left ideal of S is intuitionistic fuzzy semiprime.

Proof:

Let $A=(\mu_A,\gamma_A)$ be an intuitionistic fuzzy left ideal of S and let $a\in S$,then there exists an element x in S such that $a=xa^2$,then $\mu_A(a)=\mu_A(xa^2)\geq\mu_A(a^2)$ since S is left regular.

Also,we have $\gamma_A(a)=\gamma_A(xa^2)\leq\gamma_A(a^2)$;that is A is fuzzy semiprime.

Proposition 4.7:

Let $A=(\mu_A,\gamma_A)$ be an intuitionistic fuzzy ideal of S.If S is intra-regular,then $A=(\mu_A,\gamma_A)$ is intuitionistic fuzzy semiprime.

Proof:

Let $a\in S$.Then there exist $x,y\in S$ such that $a=xa^2y$,thus

$\mu_A(a)=\mu_A(xa^2y)\geq\mu_A(xa^2y)\geq\mu_A(a^2y)\geq\mu_A(a^2)$

and $\gamma_A(a)=\gamma_A(xa^2y)\leq\gamma_A(a^2y)\leq\gamma_A(a^2)$

Hence, $A=(\mu_A,\gamma_A)$ is intuitionistic fuzzy semiprime.

Definition 4.8[2]:

An intuitionistic fuzzy subsemigroup $A=(\mu_A,\gamma_A)$ of S is called an intuitionistic fuzzy interior ideal of S if

1. $\mu_A(xay)\geq\mu_A(a)$

2. $\gamma_A(xay)\leq\gamma_A(a)$, for all $x,y,a\in S$.

Proposition 4.9:

Let $A=(\mu_A,\gamma_A)$ be an intuitionistic fuzzy interior ideal of S.If S is an intra-regular,then $A=(\mu_A,\gamma_A)$ is intuitionistic fuzzy semiprime.

Proof:

It follows by definition 4.4 and definition 4.8 .

REFRENCES :

[1] Zadeh.L.A.,(1965),"Fuzzy Sets", Information and Control,8:338-353.

[2] Goguen J.A.,(1967),"L-Fuzzy Sets",Math.Anal.Appl.,18:145-174.

[3] Atanassov K.T.,(1986),"Intuitionistic fuzzy sets",Fuzzy sets and Systems,20:87-96.

[4] Lajos S.,(1961),"Generalized ideals in semigroup',Scta Sci.Math.,22:217-222.

[5] Kuroki N.,(1982),"Fuzzy semiprime ideals in semigroups",Fuzzy sets and Systems,8:71-79.

[6] Martines L. ,(1996),"Fuzzy modules over fuzzy rings in connection with Fuzzy ideal of Fuzzy ring ". J.Fuzzy Mathematics , 4:843-857.

[7] Roh,E.H and Lee,J.G.(2006),"intutionstic Q-fuzy subalgebras of BCK/BCI-algebras",international Mathemtical Forum,24:1167-1174.

[8] Lajos S.,(1997),"(1,2)-ideal characterizations of unions of groups",Kobe J.Math. 5:447-450.

[9] Kuroki N.,(1981),N."On fuzzy ideals and fuzzy bi-ideals in semigroups",Fuzzy sets and Systems,5:203-215.

[10] Atanassov K.T.,(1994),"New operations defined over the Intuitionistic fuzzy sets", Fuzzy sets and Systems ,61:137-142.

Some types of fuzzy ideals in semigroups

ربيع هادي جاري اكرم برزان عطار

قسم الرياضيات- كلية التربية- جامعة ذي قار

الملخص

درسنا في هذا البحث مفهوم المثالي الضبابي في شبه الزمرة وأعطينا بعض الخواص وذكرنا بعض أنواع المثاليات مثل (المنتظم وشبه الأولية والمثاليات من النوع(٢،١) و من النوع (٢،٢)) وأعطينا بعض العلاقات بينهما.