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Computation of propagation characteristics in complex plane for multilayer optical waveguide

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ABSTRACT

This paper provides a numerical simulation for determining the complex propagation constants of integrated optics waveguide. The waveguide under consideration may consist of any number of layers with complex indices due to gain and loss. A general-purpose mode solver MAPLE program has been developed to implement the transfer matrix method, and then it applied to solving the multilayer waveguide dispersion equation in complex plane. Additionally, our program can be used to determine the electromagnetic mode structure including modal power, spatial distribution, mode size parameters, and the position of the modal peak power. Therefore, all necessary parameters for a wide range of laser devices can be calculated.

Key word: Optical waveguide, Transfer matrix, Mode solver.

INTRODUCTION

Multilayer planar waveguide structures have been widely used in the implementation of variety of optical devices including distributed feedback lasers [1], TE-TM converters [2], waveguide polarizers [3], Bragg reflectors [4], directional couplers [5], antiresonance reflecting optical waveguide (ARROW) structures [6], broad-area semiconductor lasers [7], [8].

In order to optimize the performance of integrated optical devices using such waveguides, it is important to know its propagation characteristics and the transverse modal power distribution. Since, analytical solutions does not exist for such structures, one may uses either approximate methods such as the perturbation method [9], the WKB method [10], variational method [11], graphical methods [12], mode-matching method [13], and Cauchy integration method (CIM) [14]-[17] or numerical methods [18], [19] to solve the wave equation.

The perturbation method was used for lossless 5-layer structure, lossy 4-layer structure, and for a metal-clad waveguide to determine the propagation constants and the resulting propagating mode profiles. The previous technique can not be applied successfully and then can not easily be extended to multilayer structures, since their approach is analytic and the formulas involved become cumbersome.

Numerical methods, that can efficiently and accurately solve the wave equation for the propagating modes, are thus of obvious importance since they are used as a basic tool in the design technique.

Traditional numerical zero-search algorithms such as the downhill method [20], [21], Newton's method [22] and the one-dimensional scan method in the complex plane [23] needs an initial guess value close to the actual root. Therefore, these methods are not efficient and reliable, especially for a general-purpose mode solver.

Among these methods, transfer matrix method (TMM) [24], [25] is one of the primary tools for multilayer waveguide analysis. The theory of TMM can easily generate the dispersion equation for TE and TM modes supported by such structures in a straightforward manner. Multilayer waveguide can consist of any combination of lossless and lossy (dielectric, semiconductor, metallic) and active (including uniaxially anisotropic quantum well) layers. The guided mode propagation constants of the structure correspond to zeros of the dispersion equation.

A program was needed for solving the wave equation for guided modes that consist of many layers with complex refractive indices. The imaginary part of refractive index describes gain and loss in the layers. The program developed should be run on a personal computer within an acceptable time and accuracy.

In this paper, we develop a MAPLE program for the solution of the characteristic equation in the complex plane numerically corresponding to TE and TM modes of a multilayer planar optical waveguide. The TMM based program is extended to compute the mode field profile and its evolution, mode size, and its peak position. In section 2, we will discuss the necessary theory of the method, and in section 3, we will present some numerical results to show the validity of our calculations. Conclusions will drown in section 4.

Transfer Matrix method (TMM)

Maxwell's curl equations for source-free, time-harmonic fields in anisotropic media are:

$$\nabla \times \overline{E} = -j\omega\mu_o \overline{H} \tag{1a}$$

$$\nabla \times \overline{H} = j\omega\varepsilon_o\varepsilon_r \overline{E} \tag{1b}$$

Where ε_o is the free space permittivity, ω is the angular frequency, and ε_r is a relative permittivity.

Consider a planar multilayer waveguide as shown in Fig. 1. For mode propagating along the $+\hat{z}$ direction in the homogeneous *i*-th layer, the field components E_x, E_z, H_y will vanishes for TE mode, whereas H_x, H_z, E_y will vanishes for TM mode. With these assumptions the wave equation for the *i*-th layer reduces to

$$\frac{\partial^2}{\partial x^2} F_{y,i}(x) - \left(\beta^2 - k_o^2 n_i^2\right) F_{y,i}(x) = 0\,, \quad x_i \le x \le x_{i+1} \tag{2}$$

Where $F_y = \begin{cases} E_y, & TE \\ H_y, & TM \end{cases}$,

$\beta=\beta_{re}+j\beta_{im}$ is the complex propagation constant of the mode, $k_o=2\pi/\lambda$ is the free space wavenumber. The layers have complex refractive indices $n=n_{re}+jn_{im}$, where the imaginary part is due to gain or loss. The effective index n_{eff} and the absorption coefficient α_{WG} are given, respectively, by [24]

$$n_{eff}=\beta_{re}/k_o \tag{3a}$$

$$\alpha_{WG} = 2\beta_{im} \tag{3b}$$

The general solution of the wave equation in each homogeneous layer (i) is well known

$$F_{y,i}(x) = A_i \exp(\kappa_i (x - x_i)) + B_i \exp(-\kappa_i (x - x_i)) \tag{4}$$

Where $\kappa_i = \sqrt{\beta^2 - k_o^2 n_i^2}$, A_i and B_i are the complex field coefficients that vary from layer to layer, and x_i is the position of the interface between layer i and i+1. By imposing the continuity conditions of the field and its derivatives for each interface, it is easy to find [24], [26]

$$\begin{bmatrix} A_{i+1} \\ B_{i+1} \end{bmatrix} = T_i \begin{bmatrix} A_i \\ B_i \end{bmatrix} \tag{5}$$

Where

$$T_i = \frac{1}{2}\begin{bmatrix} \left(1+\eta_i \dfrac{\kappa_i}{\kappa_{i+1}}\right)\exp(\kappa_i d_i) & \left(1-\eta_i \dfrac{\kappa_i}{\kappa_{i+1}}\right)\exp(-\kappa_i d_i) \\ \left(1-\eta_i \dfrac{\kappa_i}{\kappa_{i+1}}\right)\exp(\kappa_i d_i) & \left(1+\eta_i \dfrac{\kappa_i}{\kappa_{i+1}}\right)\exp(-\kappa_i d_i) \end{bmatrix}$$

Where d_i is the i-th layer thickness and

$$\eta_i = \begin{cases} 1, & TE \\ n_{i+1}^2 / n_i^2, & TM \end{cases},$$

One can relate field coefficients in the cladding (A_c and B_c) with the coefficients in the substrate (A_s and B_s) as follows:

$$\begin{bmatrix} A_s \\ B_s \end{bmatrix} = T \begin{bmatrix} A_c \\ B_c \end{bmatrix} \tag{6}$$

Where $T = T_N \ldots T_2 T_1 T_c = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{55} \end{bmatrix}$ and N is the total number of layers.

For the guiding modes, the fields should be evanescent in the cladding and the substrate layers, so A_c=0 and B_s=0 that results the characteristic equation [24]

$$t_{11}(\beta) = 0 \tag{7}$$

For multimode waveguide, there is a z-dependent phase difference between modes due to modal dispersion. As a result the combined mode field for multimode waveguide is given by superposition of all the guided mode fields as follows [25]

$$F_{y_{total}}(x,z) = \sum_{m=1}^{M} F_{y_m}(x)\exp(-j\beta_m z) \tag{8}$$

Where *M* is the total number of modes.

Numerical Results and Discussions

A program has been developed using MAPLE symbolic computational language to handle the transfer matrix method. The built-in *floating point arithmetic* root finding method through *fsolve* function to determine the complex roots of the characteristic equation (Eq. 7) that corresponds to the propagation and loss constants directly without needing to find the analytical derivatives for the characteristic equation as required in CIM [12]-[15].

In this section, we present some numerical results to test the validity of our program. We have computed the propagation and loss characteristics, the modal field profile, the position of power maxima, and the mode size of TE and TM modes that propagates in multilayer planar optical waveguides.

Our program was applied to 6-layer lossy waveguide. The typical values of the various parameters are assumed as follows [17]: n_s=3.172951, n_1=3.16455, n_2=3.22534, n_3=3.39583, n_4=3.5321-*j*0.08817, n_5=3.39614, n_6=3.38327, n_c=1.0, d_1=0.6□*m*, d_2=1.6□*m*, d_3=0.518□*m*, d_4=0.6□*m*, d_5=0.2□*m*, d_6=0.1□*m*, □$_o$=1.523□*m*, which corresponds to the multi-mode region. The results are perfectly agree with those reported in [17] as indicated in Table 1.

The dispersion equation of the multilayer planar waveguide (Eq. 7) has been numerically solved to calculate the effective index and loss of the guided modes and plotted as a function of k_o for *TE* and *TM* modes as shown in Figs. 2 and 3, respectively. Upon normalizing the time-averaged power per unit length in the lateral direction, the field profile (Eq. 4) of each guided mode and its evolution (Eq. 8) can be obtained by successively applications of the transfer matrix to the field at the reference interface plane. Having found the roots, the coefficients $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$ of each layer can be calculated from Eq. 6. $\boldsymbol{A}_1$ is equal to zero and $\boldsymbol{B}_1$ has to be determined. Since all coefficients are proportional to $\boldsymbol{B}_1$, $\boldsymbol{B}_1$ reflects the normalization, which has been accomplished according to Ref.[25].

In Figs. 4 and 5, we have plotted the modal power profiles $\left|E_y\right|^2$ and $\left|H_y\right|^2$ as a function of x for TE and TM modes for k_o=2.7 and 4.0 □m^{-1}, respectively. Such plots are extremely useful in computing the power peak position as a function of k_o (see Fig. 6) and mode size which is defined as the value of x where the power value reduces to $1/e$ of its peak value (see Fig. 7). The modal spatial power distribution are also useful in overlap integral calculations for estimating the efficiencies in electrooptic or acoustooptic interactions using such waveguides, coupling efficiency calculations, etc.

Mode field evolution of this structure can be determined along its propagation direction using Eq. 8 and plotted for TE- and TM-modes for different values of k_o as shown in Fig. 8. The propagation constants are tabulated in Table 2. The calculations are repeated in the case of absence the imaginary part of the refractive index (lossless case) for comparison purposes and also shown in the same figure. The waveguide support three modes with effective refractive index (n_{eff}) as shown in Table 2 for the complex case and in Table 3 for the real case.

Also, Figs.8 shows that the lightwave are not coupled between the layers as it propagate along the structure because of the attenuation that it suffers due to the waveguide loss which uses lossy materials. Whereas, for lossless case, the coupling between the different modes are very effected on the power evolution. Moreover, the coupling lengths between the different propagating modes can be estimated directly from this figure.

Conclusions

In this paper, we have presented a general-purpose mode solver MAPLE program to analyze multilayer planar optical waveguide. We have used TMM and some numerical methods in the analysis. The method not only converges rapidly but is also capable of giving results of specified accuracy. The program can be used in effective index and loss, field or power profiles, and mode size calculations

Table 1. Complex propagation constants of guided modes in a six layer planar optical waveguide.

Mode	Present work		Ref. [17]	
	β_{re}/k_o	β_{im}/k_o	β_{re}/k_o	β_{im}/k_o
TE_o	3.460829693510364	− 0.072663342917385	3.460829694	− 0.07266334292
TE_1	3.3167078020463705	− 0.023275817588124	3.316707802	− 0.02327581759
TE_2	3.2085554287344547	− 0.012782067986634	3.208555428	− 0.01278206799
TE_3	3.1954905933965134	− 0.012585955654403	3.195490593	− 0.01258595565
TM_o	3.4553316045512017	− 0.070593844189186	3.455331605	− 0.07059384419
TM_1	3.3106349364087075	− 0.023388566475009	3.310634936	− 0.02338856648
TM_2	3.2080266212178024	− 0.006483752441067	3.208026621	− 0.00648375244
TM_3	3.1818980284442880	− 0.01579829719004	3.181898028	− 0.01579829719

Table 2. Complex propagation constants (β/k_o) of guided modes for different wavenumbers.

Mode	κ_o=2.7 μm^{-1}	κ_o=3.4 μm^{-1}	κ_o=4.0 μm^{-1}
TE_o	3.418808020- j 0.061935237	3.443618759 -j 0.068083975	3.458278409 -j 0.071970731
TE_1	3.231382960 -j 0.013037341	3.279635864 -j 0.018475813	3.311244455 -j 0.022355727
TE_2	3.176756803 -j 0.003507340	3.197361028 -j 0.003027743	3.207205713 -j 0.007778636
TM_o	3.404932077 -j 0.057347714	3.435062986 -j 0.065123524	3.452367984 -j 0.069785096
TM_1	3.220435918 -j 0.012377336	3.269908921 -j 0.019447936	3.304622318 -j 0.022667265
TM_2	3.171668419 -j 0.003752703	3.195644700 -j 0.003044799	3.206415970 -j 0.004951341

Table 3. Propagation constants ($\beta/k_o=n_{eff}$) of guided modes for different wave-numbers for the real case.

Mode	κ_o=2.7 μm^{-1}	κ_o=3.4 μm^{-1}	κ_o=4.0 μm^{-1}
TE_o	3.4228669810354166528	3.4474952236015813512	3.4618876371482050990
TE_1	3.2310781503658006355	3.2803754628690072894	3.3141704678749249900
TE_2	-	3.1978754361028033065	3.2117608765242057352
TM_o	3.4087200415636834068	3.4387602181249710447	3.4558038439970183340
TM_1	3.2205563130804075898	3.2698288571315257506	3.3061495419363857672
TM_2	-	3.1957087876663750962	3.2084569800733149295

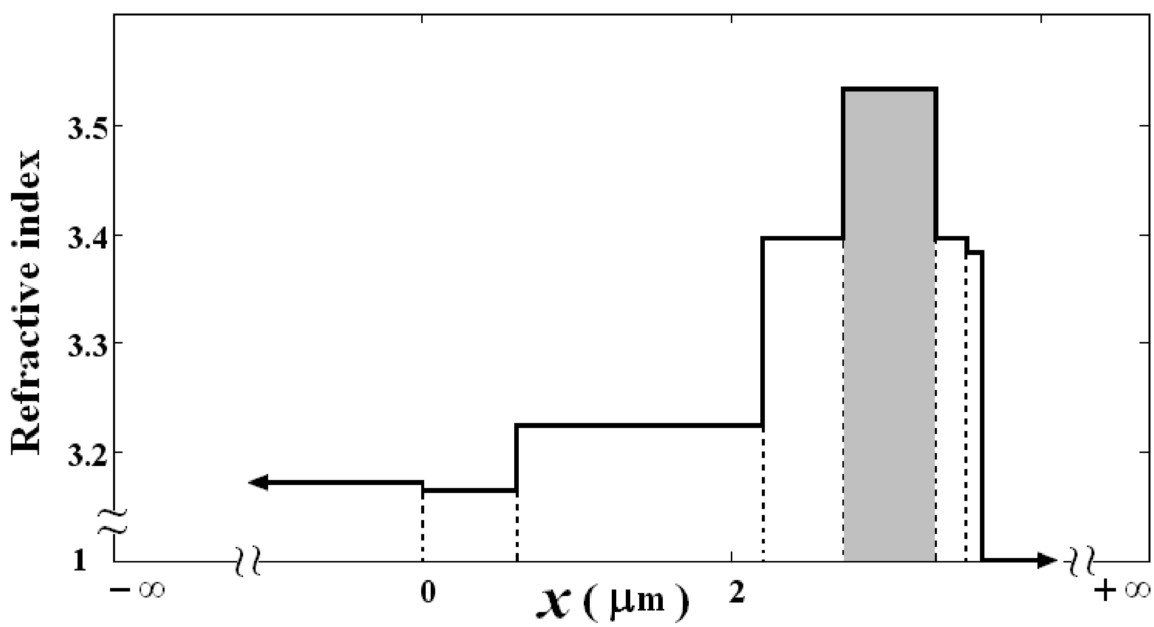

Fig.1. index profile for 6-layer planar optical waveguide.

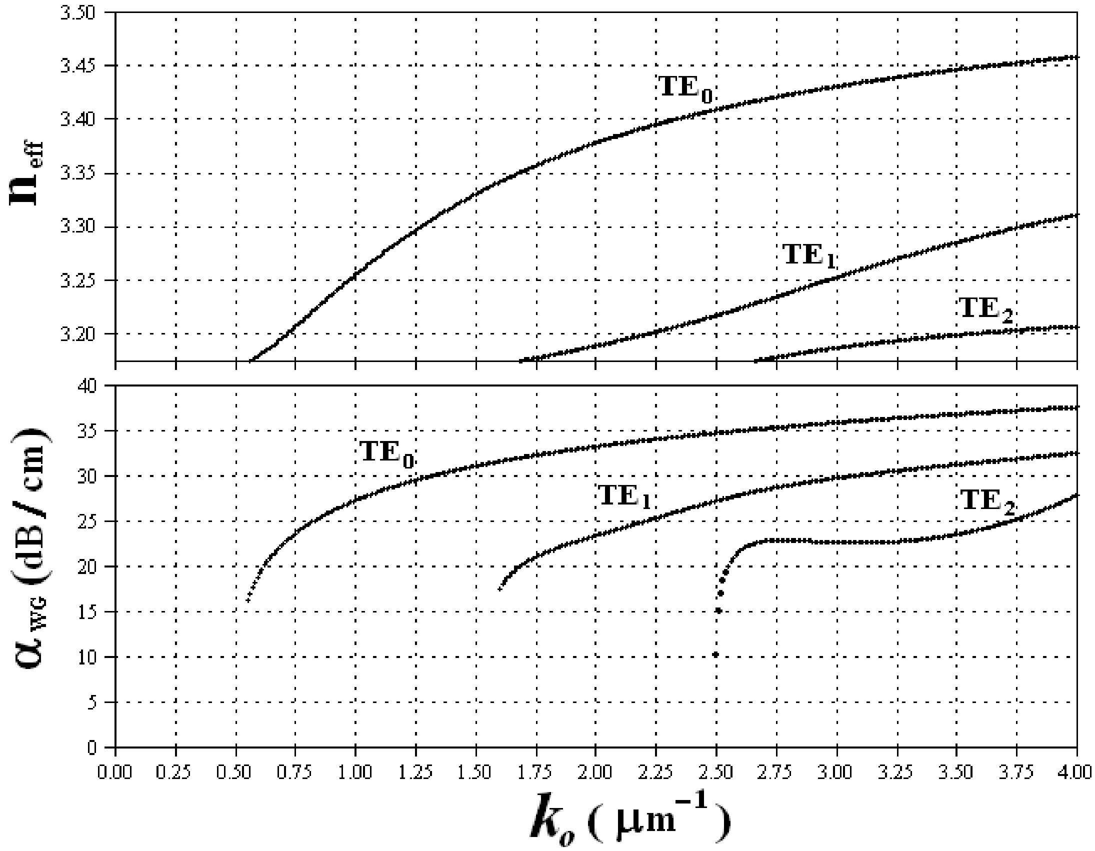

Fig. 2. Propagation and loss characteristics of 6-layer planar waveguide for TE

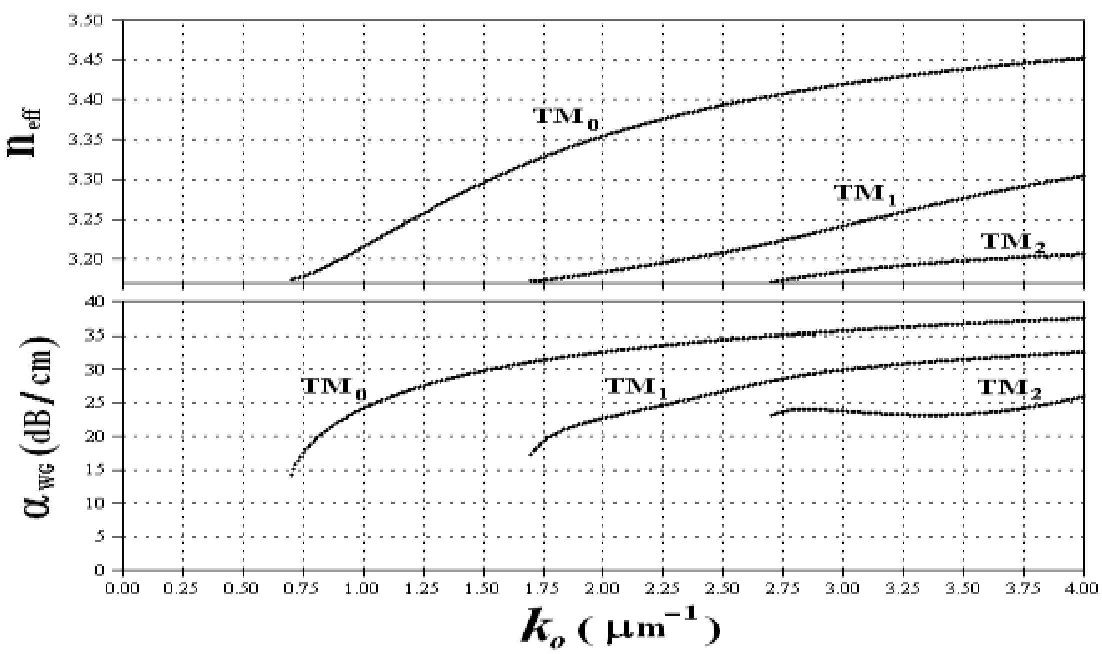

Fig. 3. Propagation and loss characteristics of 6-layer planar waveguide for TM modes

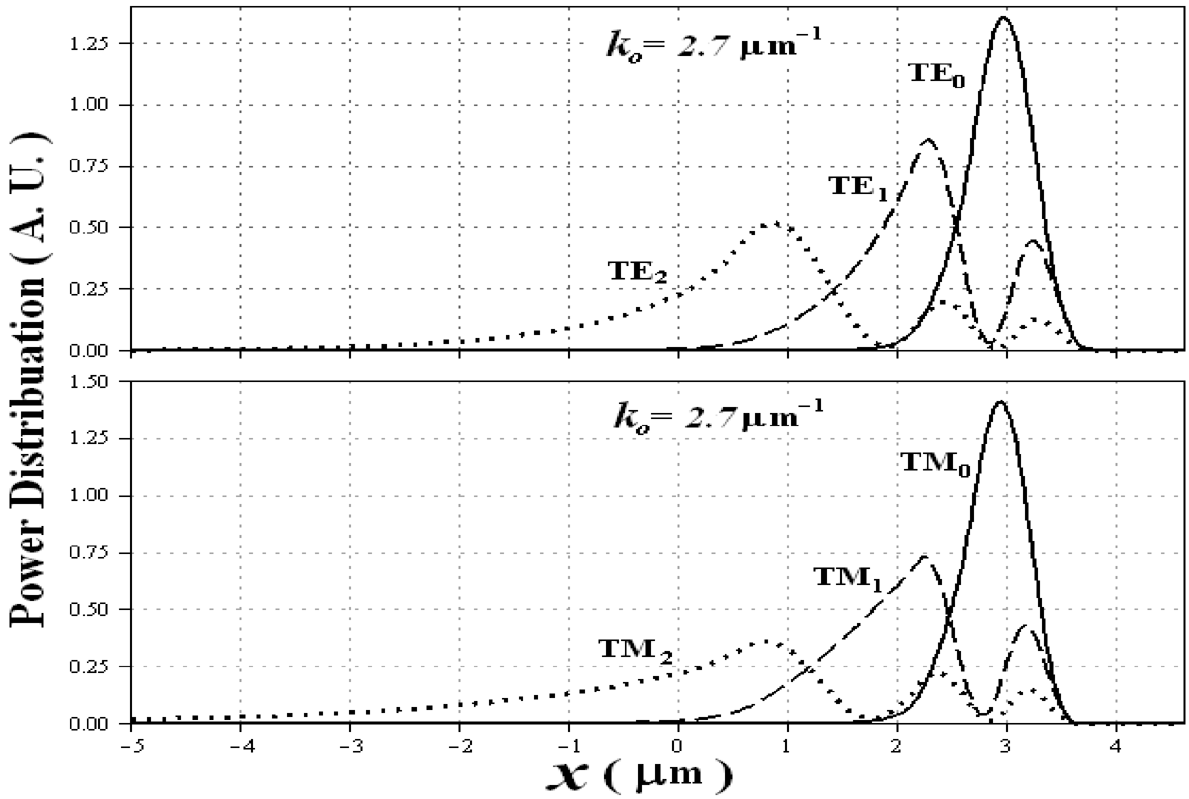

Fig. 4. Modal power distribution in 6-layer planar waveguide for TE and TM modes at k_o=2.7 μm^{-1}.

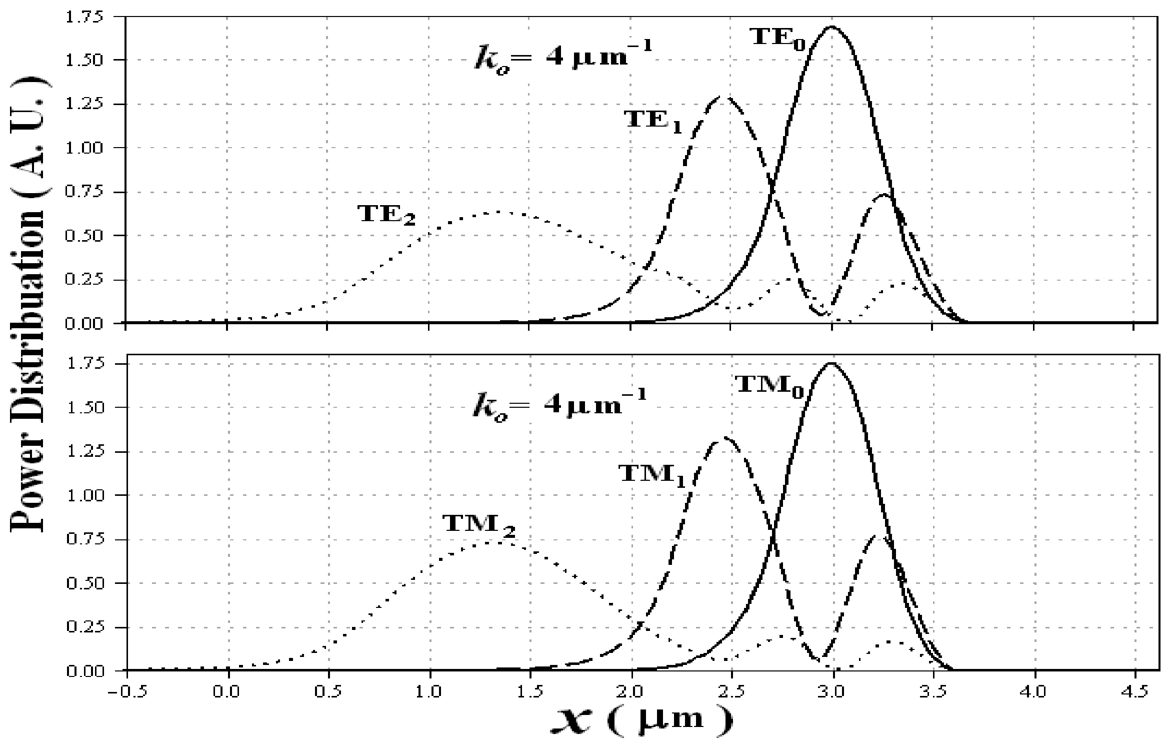

Fig. 5. Modal power distribution in 6-layer planar waveguide for TE and TM modes at k_o=4.0 μm^{-1}.

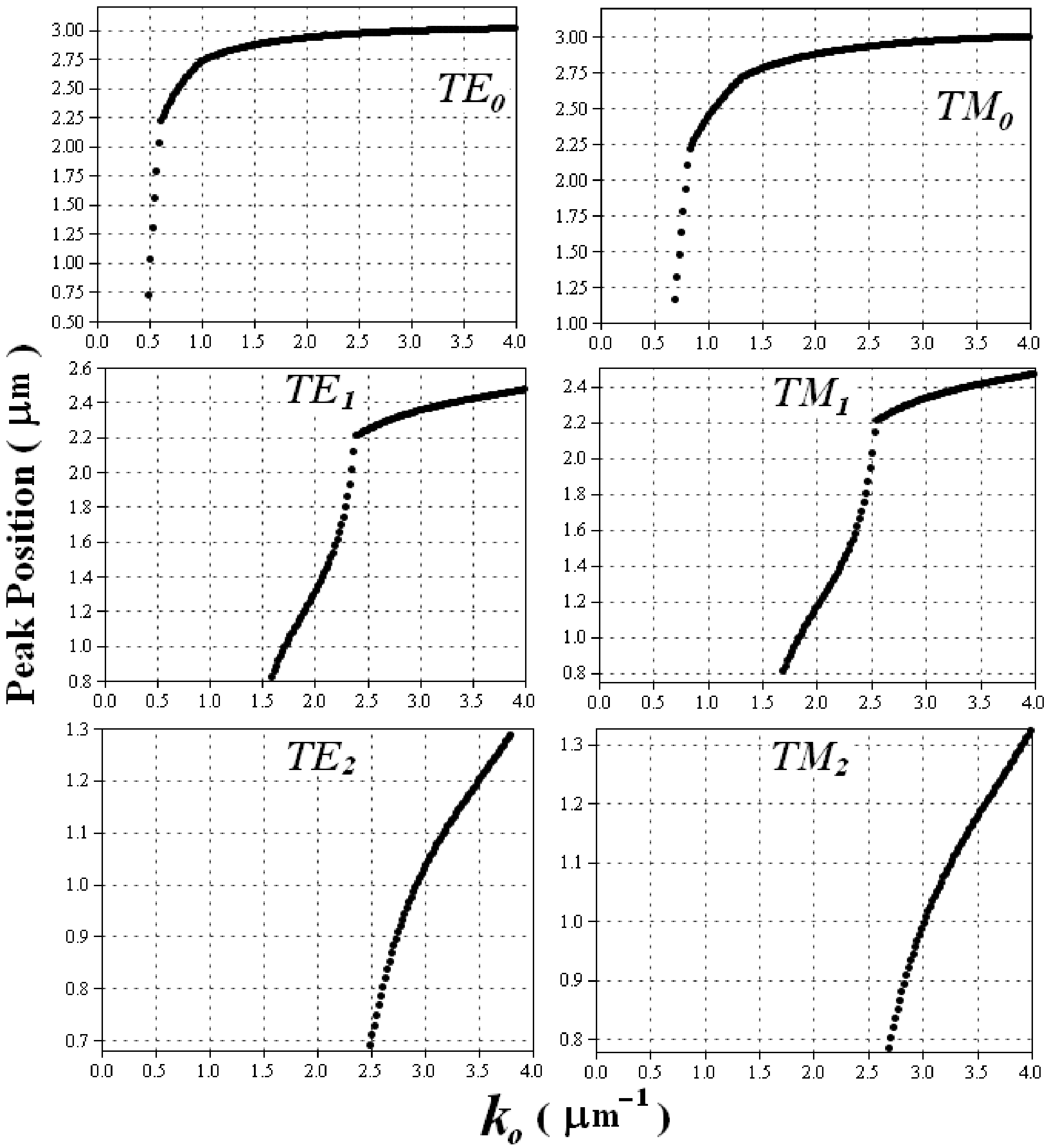

Fig. 6. Peak position of the power profile as a function of k_o for TE and TM modes.

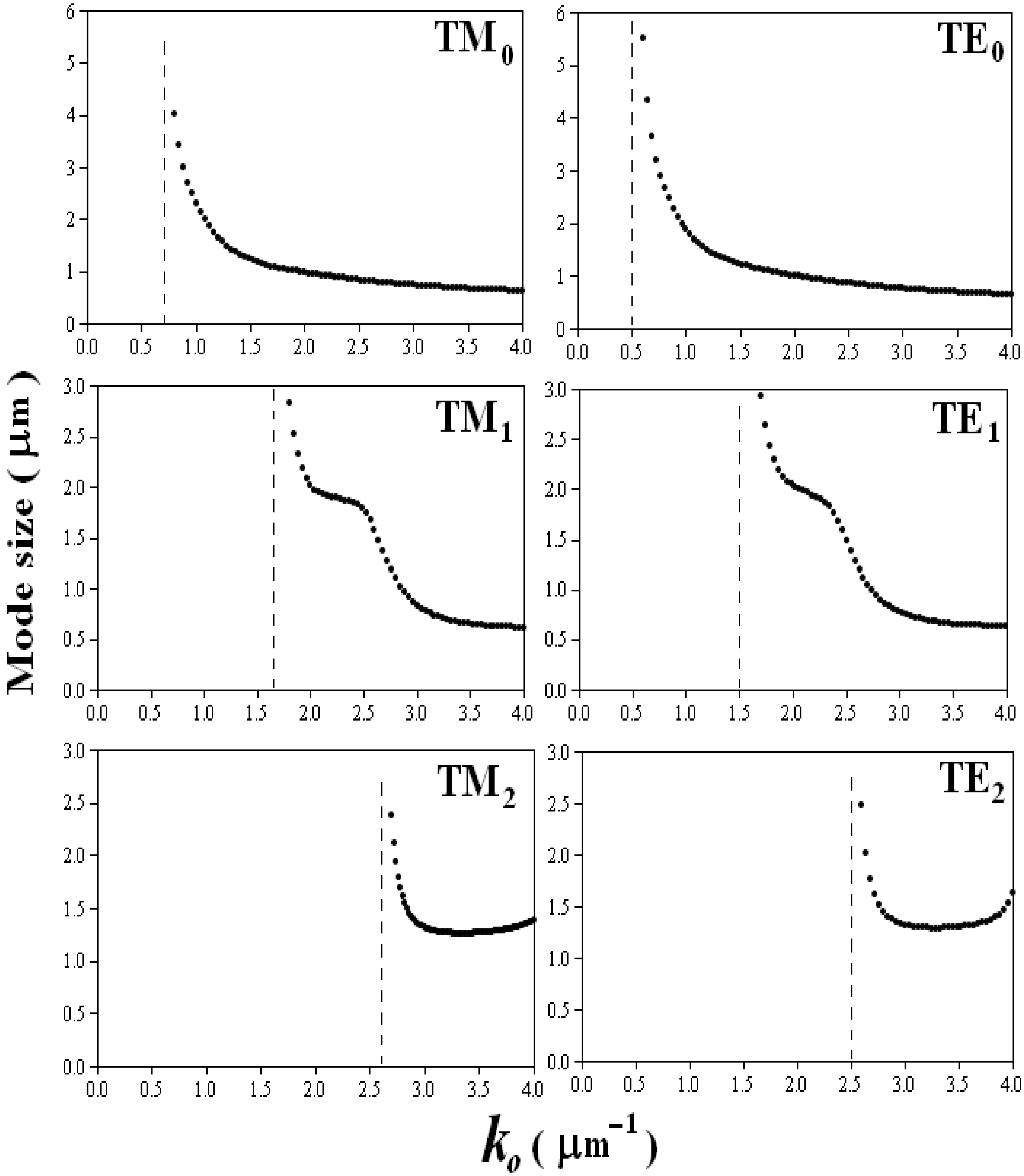

Fig. 7. Mode size as a function of k_o for TE and TM modes.

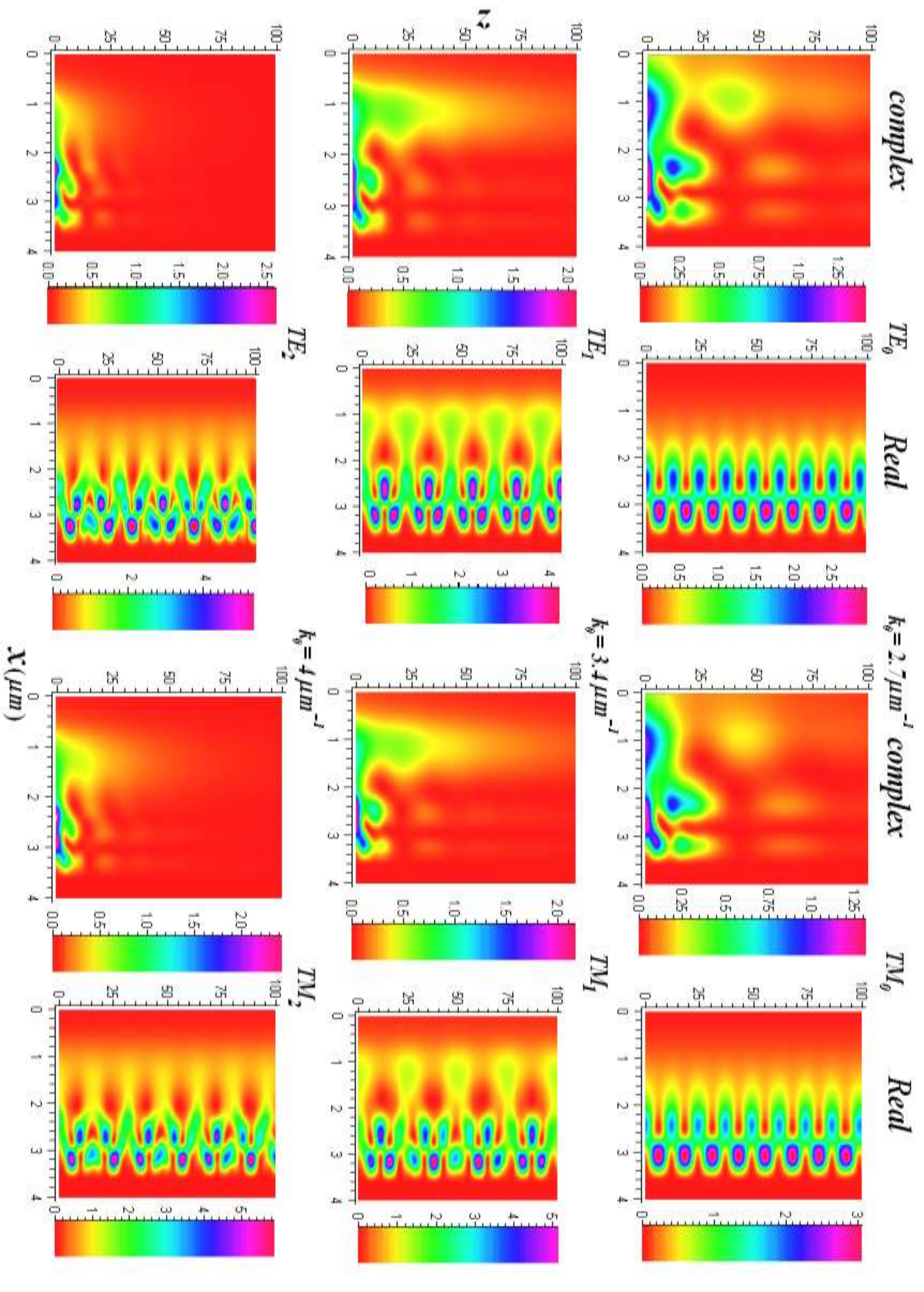

Fig. 8. The evolution of the TE and TM modes in 6-layer planar waveguide for three different values of k_o=2.7, 3.4 and 4.0 μm^{-1}. The figure is plotted for the case of complex and real propagation constant.

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حساب خواص الانتشار في مستوي عقدي لدليل موجة بصري متعدد الطبقات

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الخلاصة

تم في هذا البحث تقديم محاكاة عددية لنظام دليل الموجة البصري المتكامل لغرض حساب ثوابت الانتشار العقدية. تمتاز هذه الأدلة بانها تتألف من عدة طبقات رقيقة بمعاملات انكسار عقدية ناتجة عن الفقد أو التحصيل. تم الاستعانة بنظام MAPLE لغرض تصميم برنامج متعدد الأغراض يطبق في حل معادلة التفريق في المستوي العقدي باستعمال طريقة المصفوفة المميزة. يمكن استعمال هذا البرنامج في حساب شكل المجالات الكهرومغناطيسية للانماط المنتشرة في هذا التركيب والتي تتضمن المعاملات التالية: التوزيع النمطي للقدرة وحجم النمط والموقع المكاني لذروة القدرة. وبهذا فيمكن حساب معظم المعاملات الضرورية لمدى واسع من الأجهزة الليزرية.