

**Instabilities and Chaos in Semiconductor Laser
With Optical Injection and Feedback**

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Abstract

A numerical has been under taken to study the effects of the optical injection and the optical feedback on the nonlinear dynamical behavior of the semiconductor laser using a simple model, based on the coupled electric field and population inversion rate equations. A number of distinct states are obtained. These include periodic limit – cycle operation , quasiperiodicity, and chaotic behaviors dependent on the type of the laser control parameter and the operating conditions.

Keywords: Instabilities, Chaos, Period – doubling, Semiconductor lasers.

Introduction

Investigations of the nonlinear dynamics of the semiconductor lasers have received increasing attention. It has been found that these lasers are very sensitive to the optical injection or the optical feedback. They are known to undergo significant changes in their output characteristics due to the strong nonlinear interaction between the optical injected field or the optical feedback signal and laser gain medium. A very rich variety of nonlinear dynamics that are of interest of many significant applications can be arisen in the semiconductor laser as a result of this interaction process. These include sources and beams for the fields of spectroscopy [1,2] , optical signal processing, and secure optical communications [3– 6].

However, over a range of operations, the nonlinear response of the semiconductor laser system to the external optical perturbations, such as modulation of injection current or the introduction of optical feedback into semiconductor laser system can induce a number of distinct dynamical nonlinear phenomena, depending on the system operating parameters and the levels of current modulation or feedback. These include parametric oscillations, four – wave mixing, and instabilities and chaotic behavior [7 – 9].

Operating semiconductor laser in injection locked state or with the presence of feedback signal has been found to have many useful advantages which are required for spectroscopic applications. These include reduction of laser line width and noise [10,11] , enhancement of signal – mode tenability and frequency stability [12–15], and modulation suppression [16,17].

For optical communication using chaotic current waveforms, it has been found that the optical injection or the optical feedback may enhance the modulation bandwidth of the semiconductor laser operating on a chaotic state regime. This allows the possibility of high bit-rate data communication [18 , 19].

Owing to the ease of operation of semiconductor lasers, it is of considerable interest to develop the semiconductor lasers with chaotic output so as to utilize them as transmitters and receivers for optical communication system. Recently , it has been shown that such system can be implemented [20 , 21].

The demonstration of single encoding / decoding at high frequency by using the semiconductor laser is the indication of suitability of this system for the optical communication. This may open new possibilities for generating an optical device with controllable instabilities and chaos, operating under the realistic experimental conditions.

The current paper presents numerical investigations into the nonlinear dynamical behavior of a semiconductor laser subject to optical injection or optical feedback.

The Semiconductor Laser Model

The nonlinear dynamics of single-mode semiconductor laser with optical injection and optical feedback can be described by the following coupled dimensionless rate equations [24]:

$$\dot{E}(t) = k(1 + \alpha)(N(t) - 1)E(t) + \eta E(t - \tau) \exp(-i\Theta) + i\Delta\omega E(t) + E_{inj} \quad (1)$$

$$\dot{N}(t) = J - N(t) - N(t)|E(t)|^2 \quad (2)$$

When $E(t)$ is the complex optical field amplitude of the laser cavity field and $N(t)$ is the normalized population inversion of the semiconductor laser. k is the cavity field decay rate, α is the line width enhancement factor, which is related to the medium refractive index, η is the dimensionless feedback parameter, τ is the feedback delay time, Θ is the phase shift, ω is the detuning between the frequency of free-running oscillation frequency and the frequency of injected signal, E_{inj} is the amplitude of the injected field, and J is the normalized injection current.

Results and Discussion

The dynamical behavior of the semiconductor laser can be described with respect to following three most effective control parameters, The field injection current (J), the cavity field decay rate (k), and the feedback rate parameter (η). We numerically solved equations (1) and (2) using the fourth-order Runge-Kutta for the above parameters.

First we have study the case in which the current modulated laser is subjected only to optical injection (in the absence of feedback, i.e., $\eta = 0$). For the laser system, we consider the following control parameters values: $E_{inj} = 0.8$, $\alpha = 2$, $k = 1$, $\Theta = 2$, and $\omega = 0.25$, although the observed features of dynamics occur over a wide parameter

range.

In Fig.1, our results shown as the effect of variation of the injection current (J) on the dynamical behavior of the semiconductor laser. The left column represents the relative laser output intensity as a function of time and the corresponding power spectrum, calculated by taking the fast-Fourier-transformation (FFT). While the right column represents the phase-space portraits corresponding to the time-series in the left column. Two of phase-space portraits are plotted, these are: The laser output intensity versus the population inversion and the imaginary part of the electric field ($\text{Im}(E)$) versus the real part of the electric field ($\text{Re}(E)$).

In Fig.1 A (a), the laser system is in a regular oscillations state with $J=1.5$. The time series shows a sequence of regular pulses with a constant intensity and interval, which is a type of stable oscillations of period-one. The corresponding power spectrum has only one fundamental frequency which is shown in Fig.1 A (b) as a single peak. The phase-space portrait is a single period-one orbit (Figs.1 A (c and d)), represents the single limit-cycle [25]. When the injection current is increased to $J = 1.65$, the laser system enters period – two oscillations state, as shown in Fig.1 B (a). The time series clearly shows the modulation of peak intensities.

The corresponding power spectrum in Fig.1 B (b) shows two frequency peaks,

one of them is too small because of the difference in the intensity between the successive pulses in the time series plot (Fig.1 A (a)). The trajectory of the system here is a period-two orbit consists of two loops as shown in Figs.1 B (c and d). When the injection current is further increased to $J = 1.90$, the laser system enters a period-four oscillations state, as shown in Fig.1 C (a). The corresponding power spectrum now has four peaks (Fig.1 C (b)), and the attractor trajectory consists of four loops as shown in Fig.1 C (c and d). This period-doubling sequences starts to convert first to quasi-periodic oscillations state with modulated pulsing intensities when the value of J is increased to $J = 1.95$ as shown in Fig.1 (D), and then to a broadband chaotic state when the value of J reaches to 2.4 (Fig.1 (E)). In the second case (when the laser system enters into a chaotic regime), it can be seen that the oscillations intensity in the time series train becomes irregular and the pulses come close to each other, that is the separation between the adjacent pulses becomes much smaller. Here, the laser system begins to lose its stability and appears to undergo complicated structure, as shown in Fig.1 E (a). This behavior can be easily identified in the corresponding power spectrum in Fig.1 E (b), where it is clearly evident that the spectrum becomes denser and highly complicated, resulting a broadband spectrum, a characteristic of chaos. This behavior can also be recognized from the spread pattern of the output trajectory attractor (Fig.1 E (c and d)), where it is seen that the trajectory is consisted of large numbers of folded loops that never intersect with each other, which is a kind of strange attractor [26]. The laser system remains at a chaotic state as the value of injection current continues to increase until abruptly changes to period – three oscillations state when the injection

current value reaches a certain level ($J = 3.0$), as shown in Fig.1 (F) , and such behavior is reported in a number of nonlinear dynamical systems [26,27]. We find that this behavior sustains over the whole range $J = 3.0 - 3.5$. As the value of J increases, the laser system gradually starts return to its starting state through reverse bifurcations route , as shown in Figs.1 (G) and 1(H) , when $J = 3.5$ and $J = 4.0$, respectively. In Fig.1 (G), we see a period – two behavior, while Fig.1 (H) shows a period – one behavior.

To study the effect of variation the filed cavity decay rate (k) on the dynamical behavior of the injected semiconductor laser system , we consider a fixed injection current value of $J = 2$. The value of k is varied over the range $k = 1.20 - 0.005$.

Representative example of the behavior of injected semiconductor laser is shown in Fig.2, which is a sequence of reverse period-doubling transitions starts with damping intensity oscillations and terminates by appearing of the chaotic behavior followed by stable period – five oscillations.

When $k = 1.2$, the laser system displays chaotic behavior as shown in Fig.2 A (a) for the intensity time series and in Fig.2 A (b) for the corresponding broadband power spectrum . this behavior id reflected in the shape of the trajectory of the output attractor as shown in Figs.2 A (c and d), and again it is a type of strange attractor.

Decreasing the value of k to 0.85 leads to period – five oscillations state as illustrated in Fig.2 B (a-d). This behavior converts into period – four oscillations state when k varies to 0.7 (Fig.2 C (a)). The power spectrum corresponding to the intensity time series is displayed in Fig.2 C (b). The system trajectory of four limit – cycle orbit, as shown in Fig.2 C (c and d).

As k is decreased to 0.65, period – two oscillations state appears, as shown in Fig.2 D (a), and two distinct peaks are clearly seen in the corresponding power spectrum, as shown in Fig.2 D (b). In this case the trajectory is a period – two attractor (Fig.2 D (c and d)). This behavior changes again to period – one oscillations when the value of k is reduced to 0.54 as shown in Fig.2 (E). We find that the continuous decreased of k value leads to sustained damped oscillations (when $k = 0.005$), as shown in fig.2 (F). The further increase of k value does not have much effect on the dynamics of the laser system, expect the variation in the output intensity. As we can see in Fig.2 (F), the intensity of the oscillations in the time series plot is reduced with time (Fig.2 F (a)), and no significant frequency peak in the power spectrum (Fig.2 F (b)). The system trajectory in this case is a fixed-point attractor as shown in Figs.2 F (c and d).

Next, we consider the case in which the semiconductor laser is subjected only to optical feedback ($E_{inj} = 0$), and the feedback rate (η) is the free parameter of our study. Different feedback levels (different η values) are considered. The results of this investigation are summarized in Fig.3 It is clear that the laser system here follows the period – doubling route into chaotic state. The dynamical behavior is a sequence of period – doubling transitions develop into regular period –three oscillations state which is immediately backs to chaotic state as the value of η further increases, without passing through additional transitions. The sequence starts with the appearance of output with chaotic oscillations when $\eta = 0.01$ (Fig.3 A (a-d)). When the value of η increases to 0.06, the laser system enters into the bifurcation region. Fig.3 B (a) reveals a

type of period-two oscillations. The corresponding power spectrum as in the preceding case has two distinct peaks as illustrated in Fig.3 B (b), while the attractor trajectory consists of two limit cycle (Figs.3 B (c and d)). The time series train bifurcates into period – four oscillations when value of η is increased to 0.12, as shown in Fig.3 (C). As the feedback strength is further increased to $\eta = 0.14$, the laser system starts exhibiting chaotic output as shown in Fig.3 D (a). This behavior is clearly evident in the corresponding power spectrum in Fig.3 D (b), and also in the attractor trajectory in Fig.3 D (c and d).

As η increases to $\eta = 0.22$, the behavior changes to period – three intensity oscillations as shown in Fig.3 (E), then the laser system jumps again into another chaotic state region when η slightly increases ($\eta = 0.225$), as shown in Figs.3 F (a–d), and remains there even with increasing the feedback to high level (high η). It is interesting to note that the dynamical behavior of period – three oscillations (Fig.3 (E)) is consistent with the general findings that the period – three state appearing in a small window within or close to the chaotic regime and it is very sensitive to the parameter variation [26].

Conclusions

We have numerically studied the dynamical behavior of a semiconductor laser subject to optical injection and feedback as modeled by simple rate equations. There are number of variable parameters in this laser system. Varying three of these parameters, the injection current, the filed cavity decay rate, and the feedback parameter, a number of distinct dynamical states have been obtained, dependent on the type and

the value of the control parameter which is varied. In general, we found that our laser system follows a period-doubling route to chaos [25]. Periodic limit-cycle operation and quasiperiodicity leads to chaos are obtained.

We note that, a state of period – three oscillations (in the case of variation J) or a state of period – five oscillations (in the case of variation k) exists within the period – doubling bifurcation. While in the case of variation η , the period – three state is located in a small interval between two chaotic states. We have found in all these case that, the chaotic state is always followed by one of these two regimes (i.e., period – three states or period – five state). This means (as we expected) that the period – three states or period – five state appears close to or within the chaotic regime. We have also found, in the case of variation k , that the dynamical behavior is always terminated by the appearance of sustained damped oscillations, with fixed-point attractor.

Useful methods for characterization the dynamical behavior of the laser system, the fast-Fourier transform and the phase-space portrait, have been used. The resulting power spectra and phase – space portraits are in good agreement with that generated from the output intensity versus time.

Using these methods enables the different dynamical states to be readily identified for more careful study.

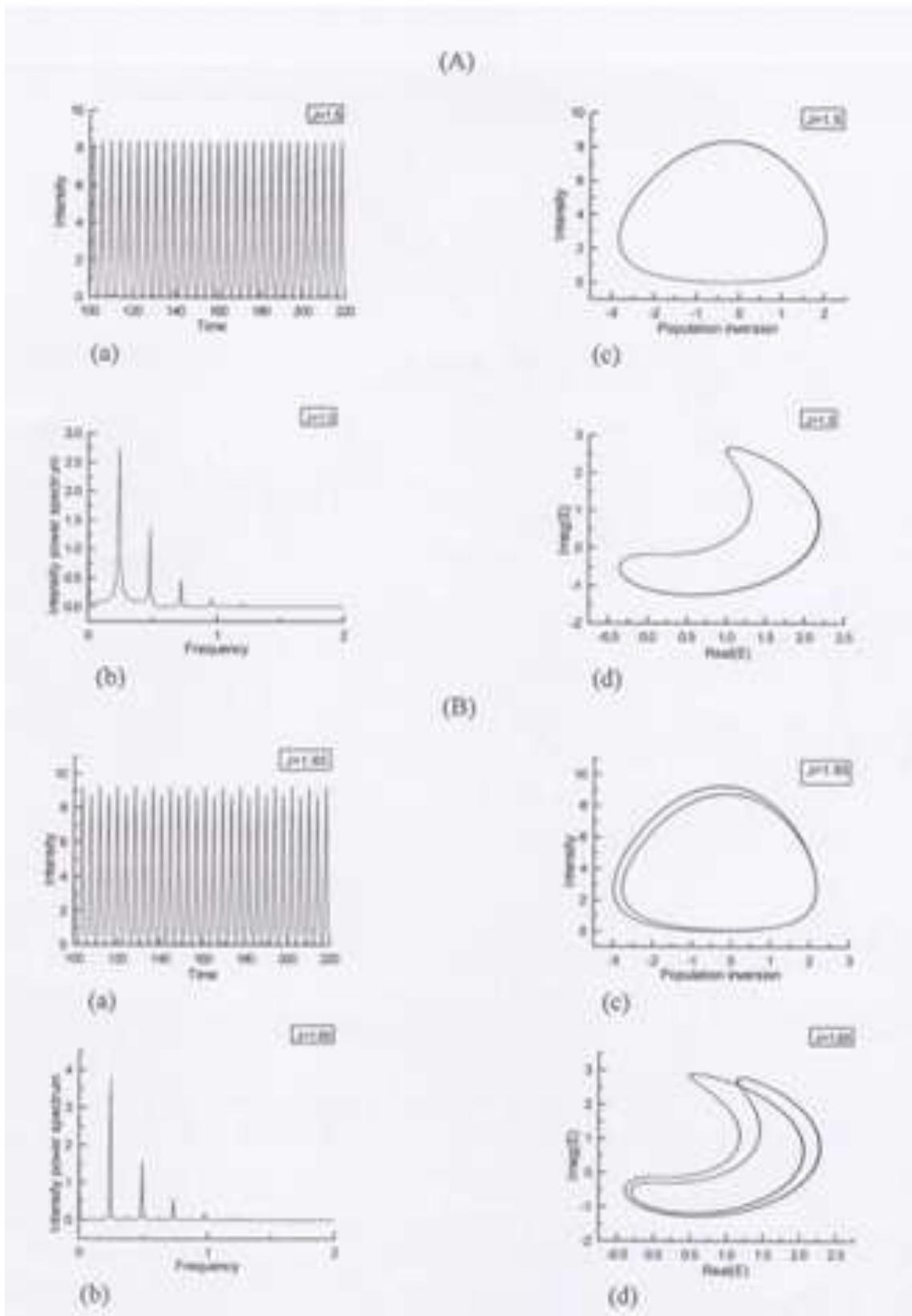
Our results indicate that the variation of the laser parameters leads to a good and nice control for this system over the whole ranges of the operating conditions.

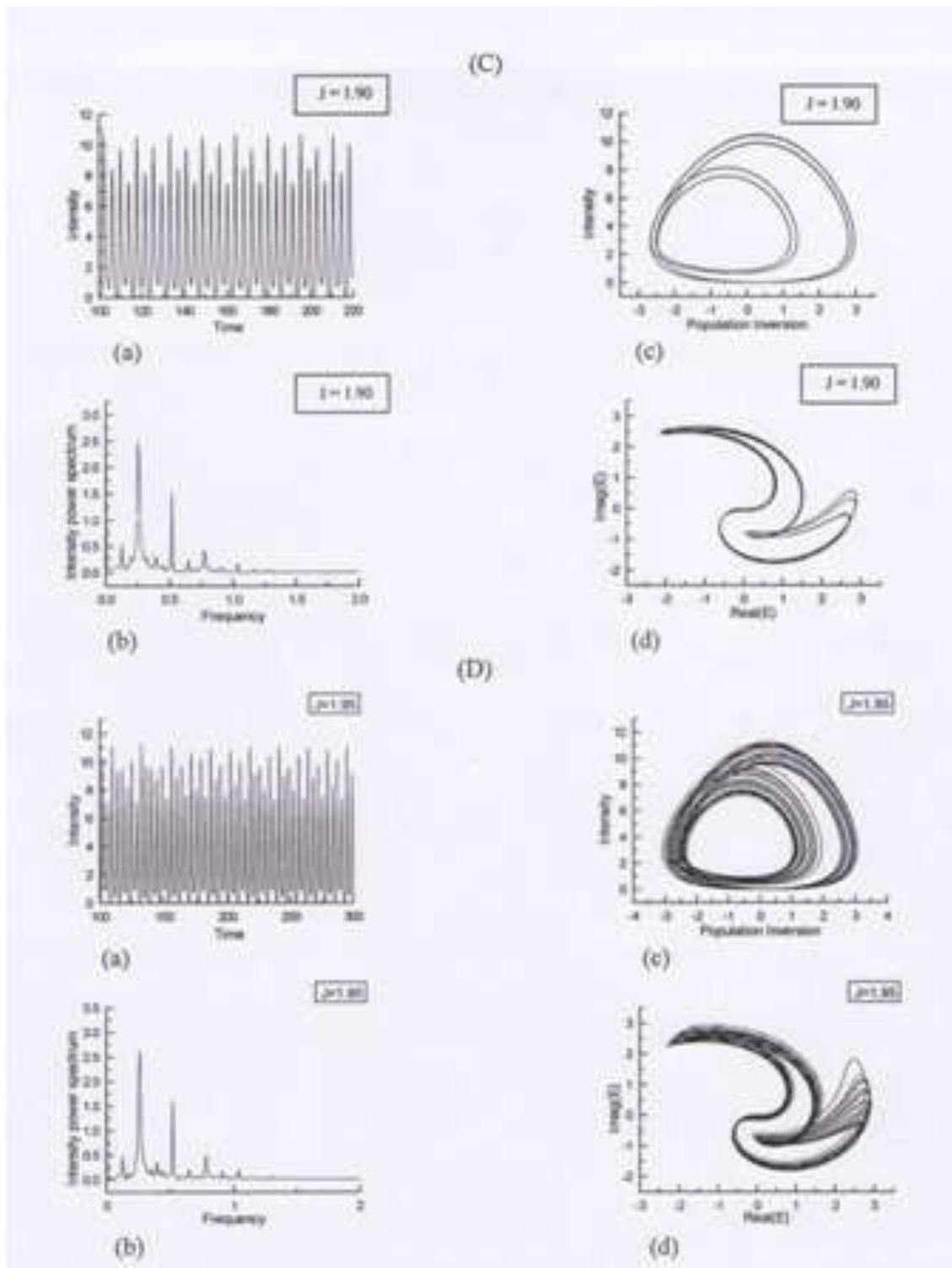
Figure Captions

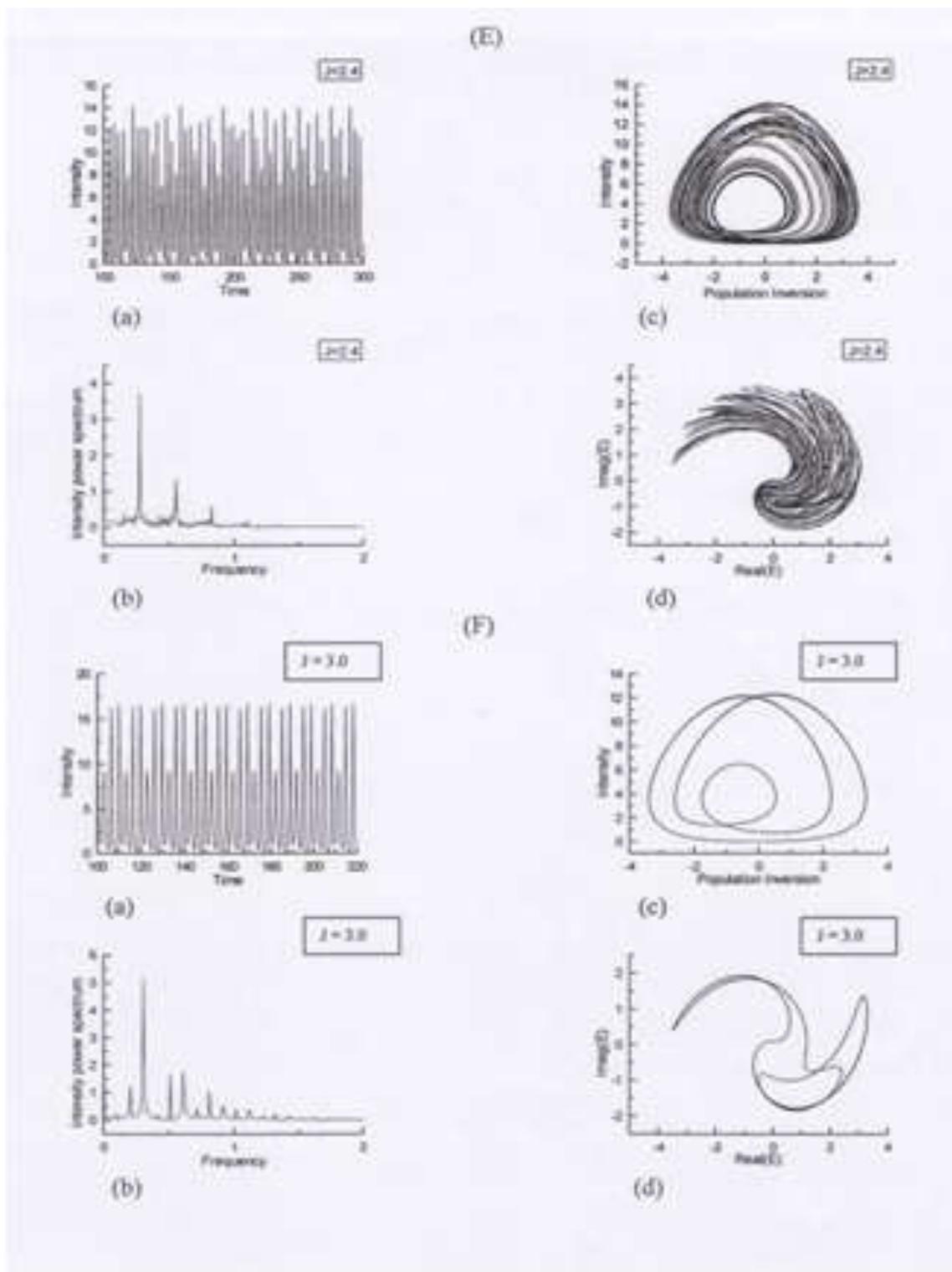
Fig.1. (a) Time series , (b) power spectrum , (c) phase-space portrait (laser output intensity versus population inversion) , and (d) phase-space portrait (electric field imaginary part versus electric field real part). For different values of J , when $\eta = 0$, $E_{inj} = 0.8$, $a = 2$, $k = 1$, $\Theta = 2$, and $\omega = 0.25$.

Fig.2. As Fig.1 but for different values of K , when $J = 2$.

Fig.3. As Fig.1 but for different values of η , when $J = 2$.







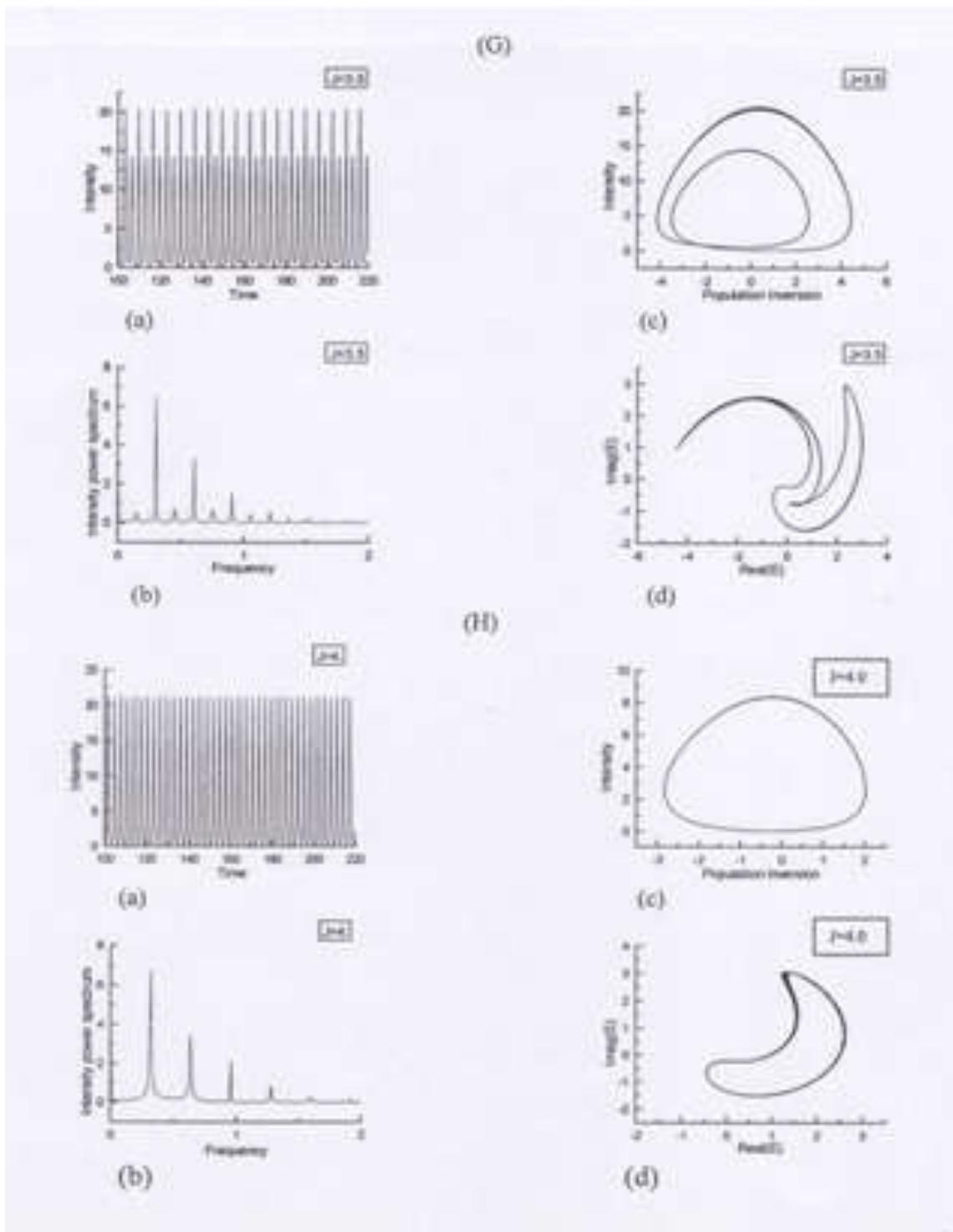
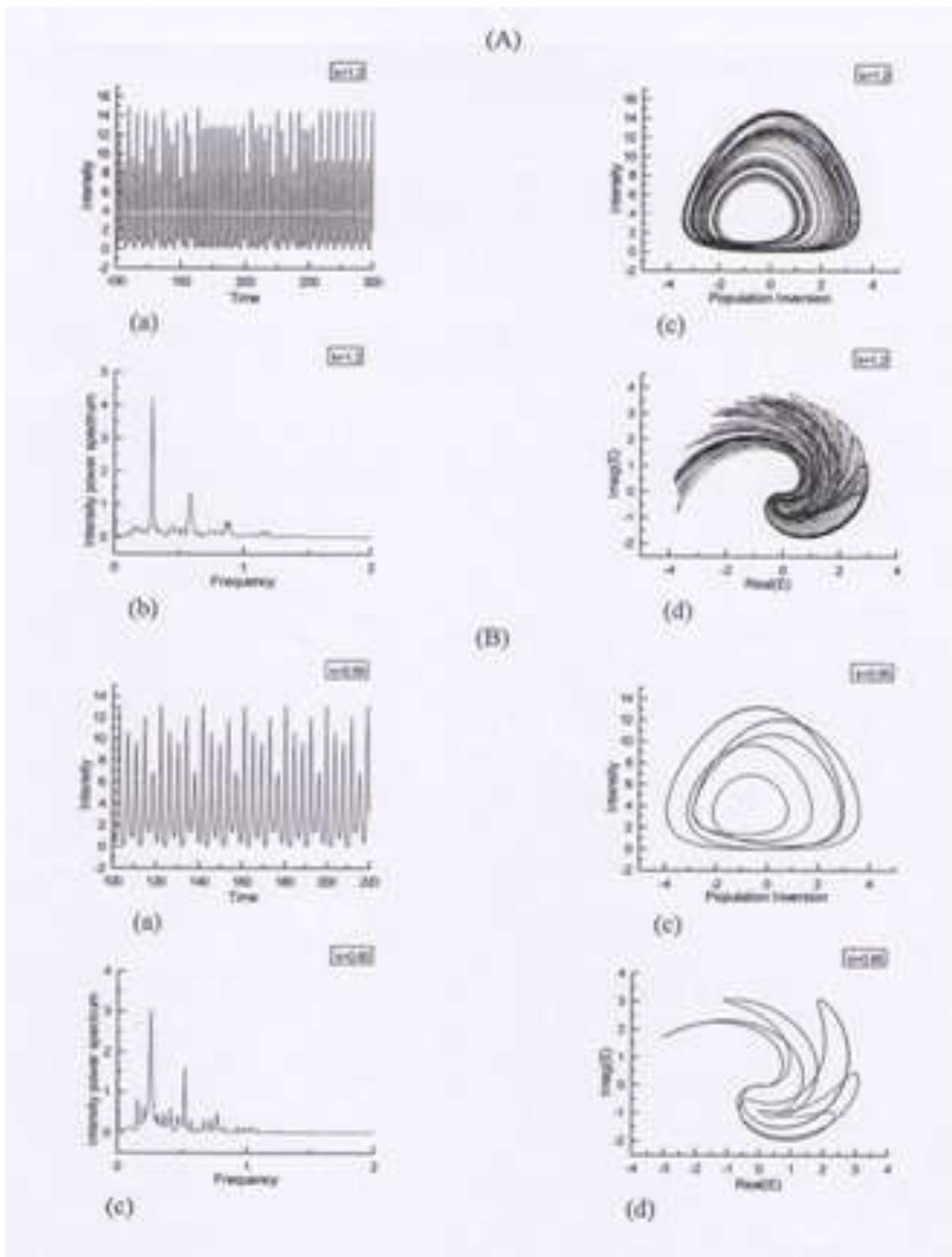
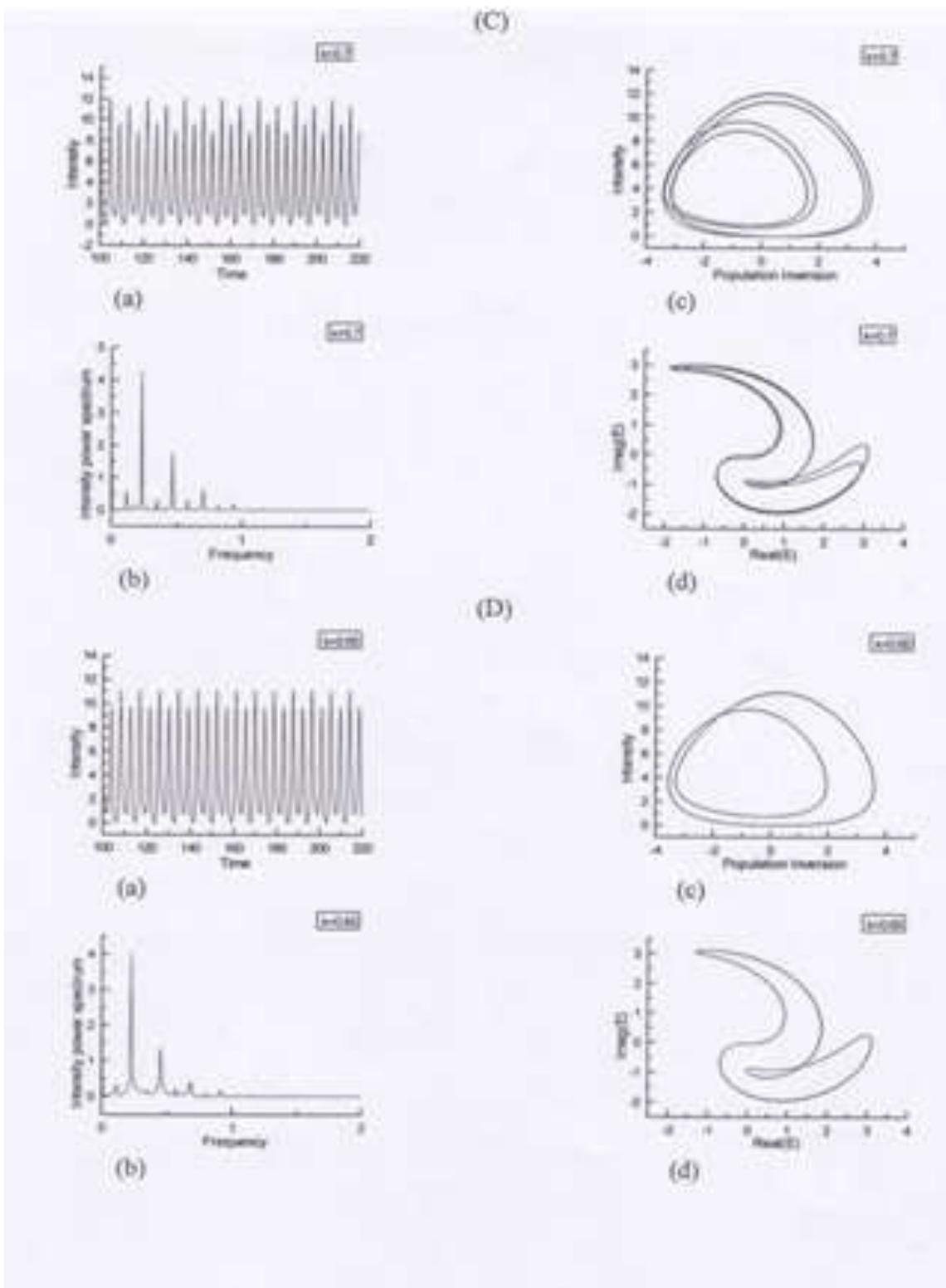


Fig.1





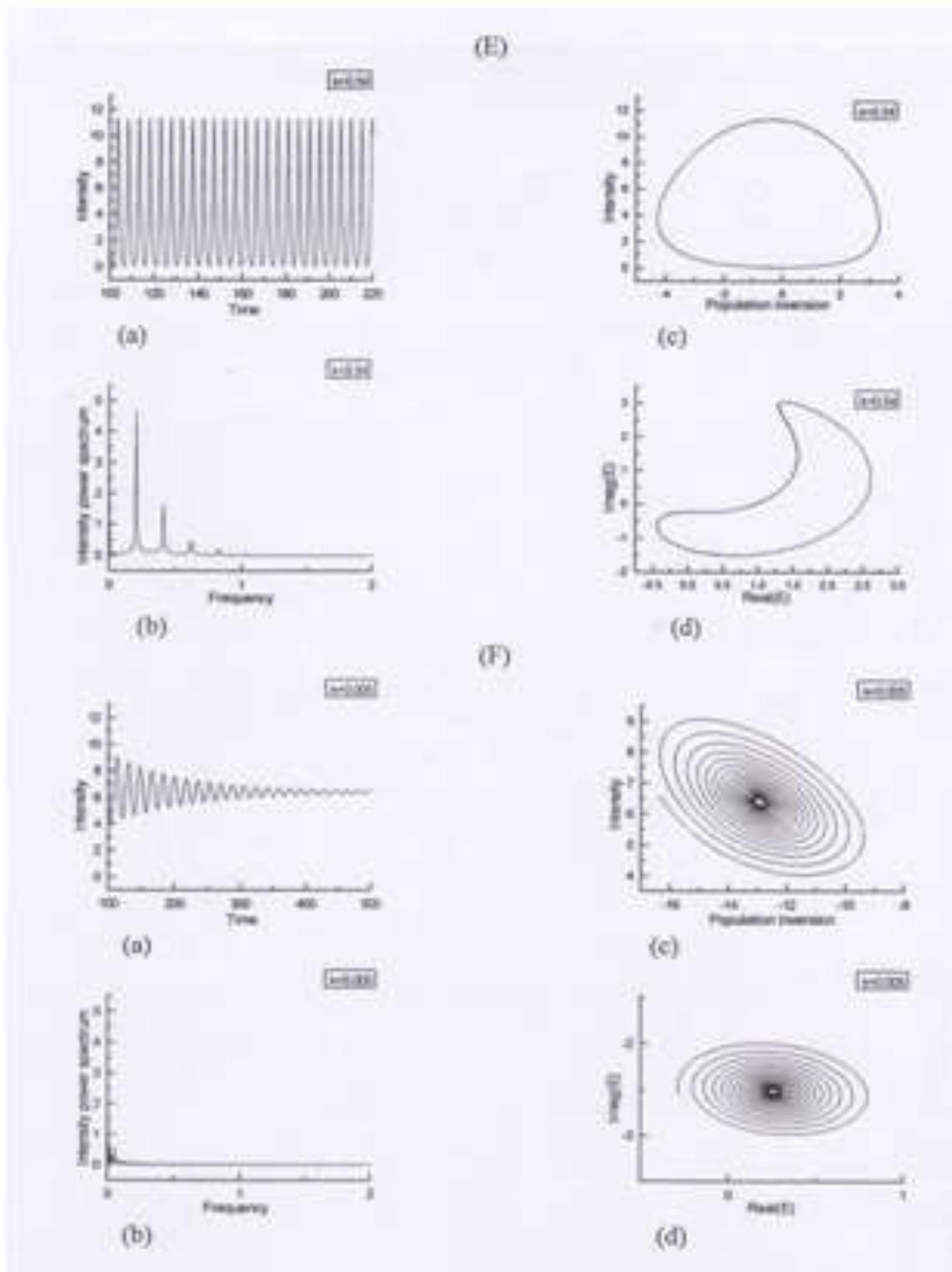
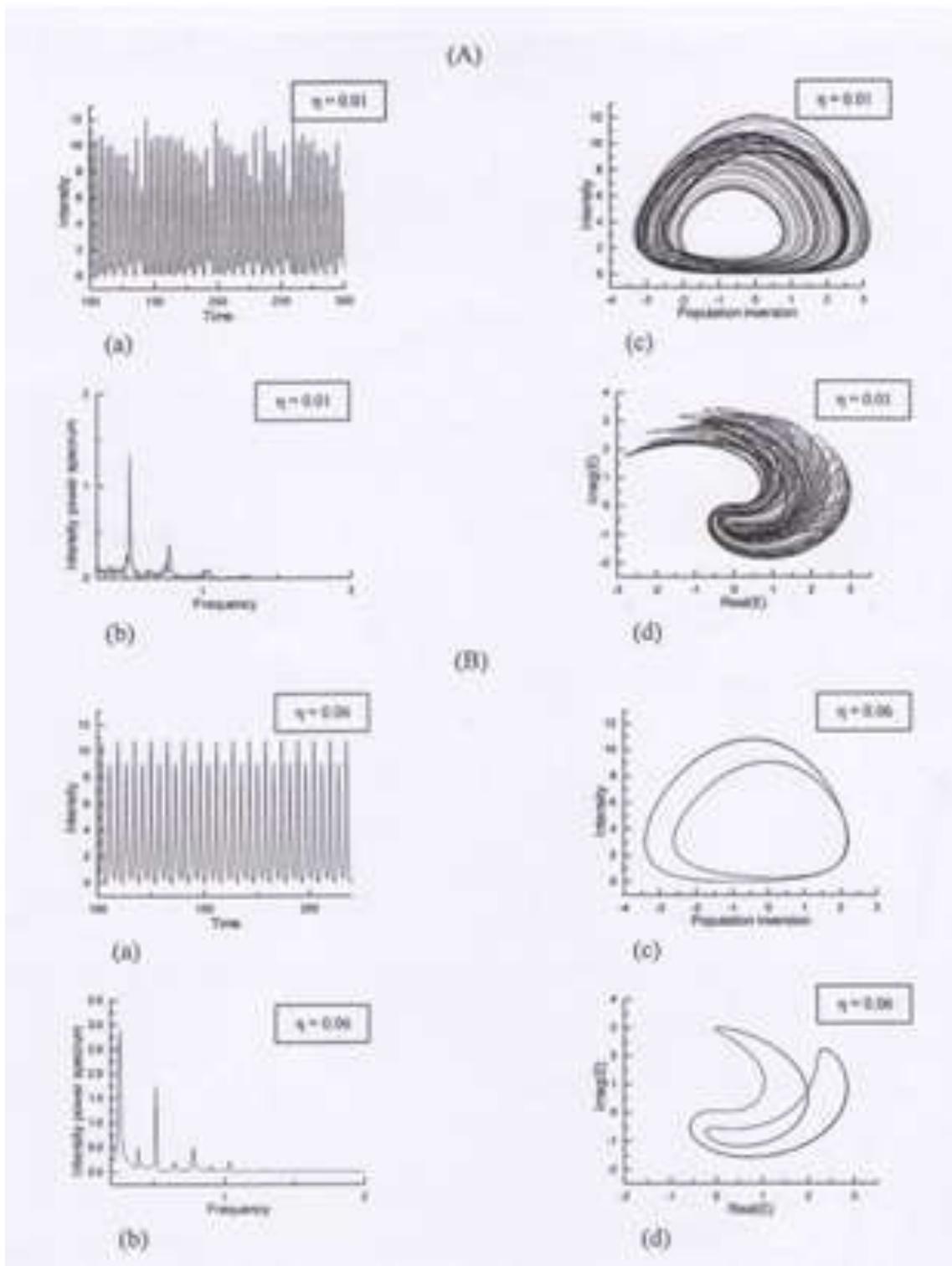
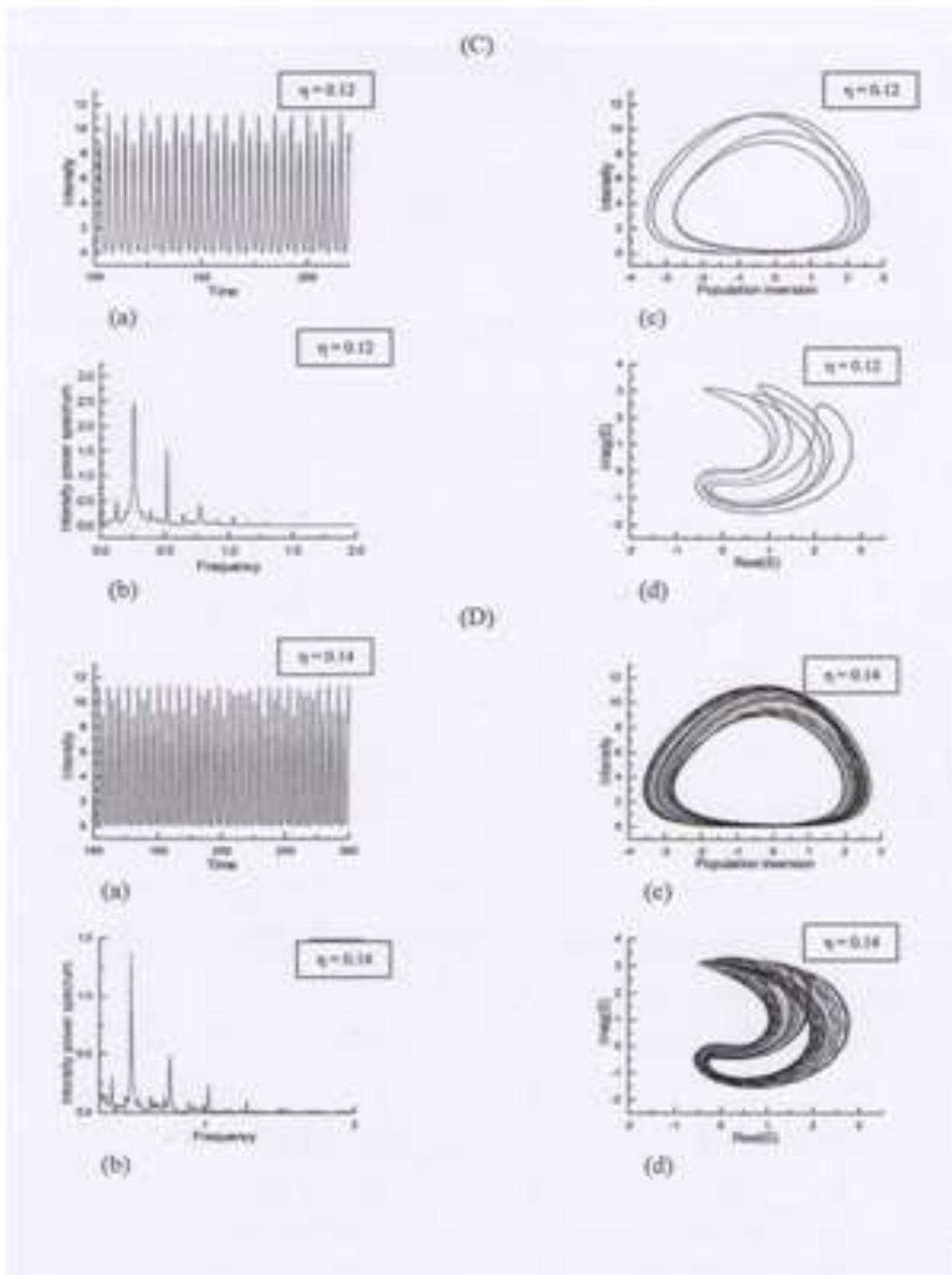


Fig.2





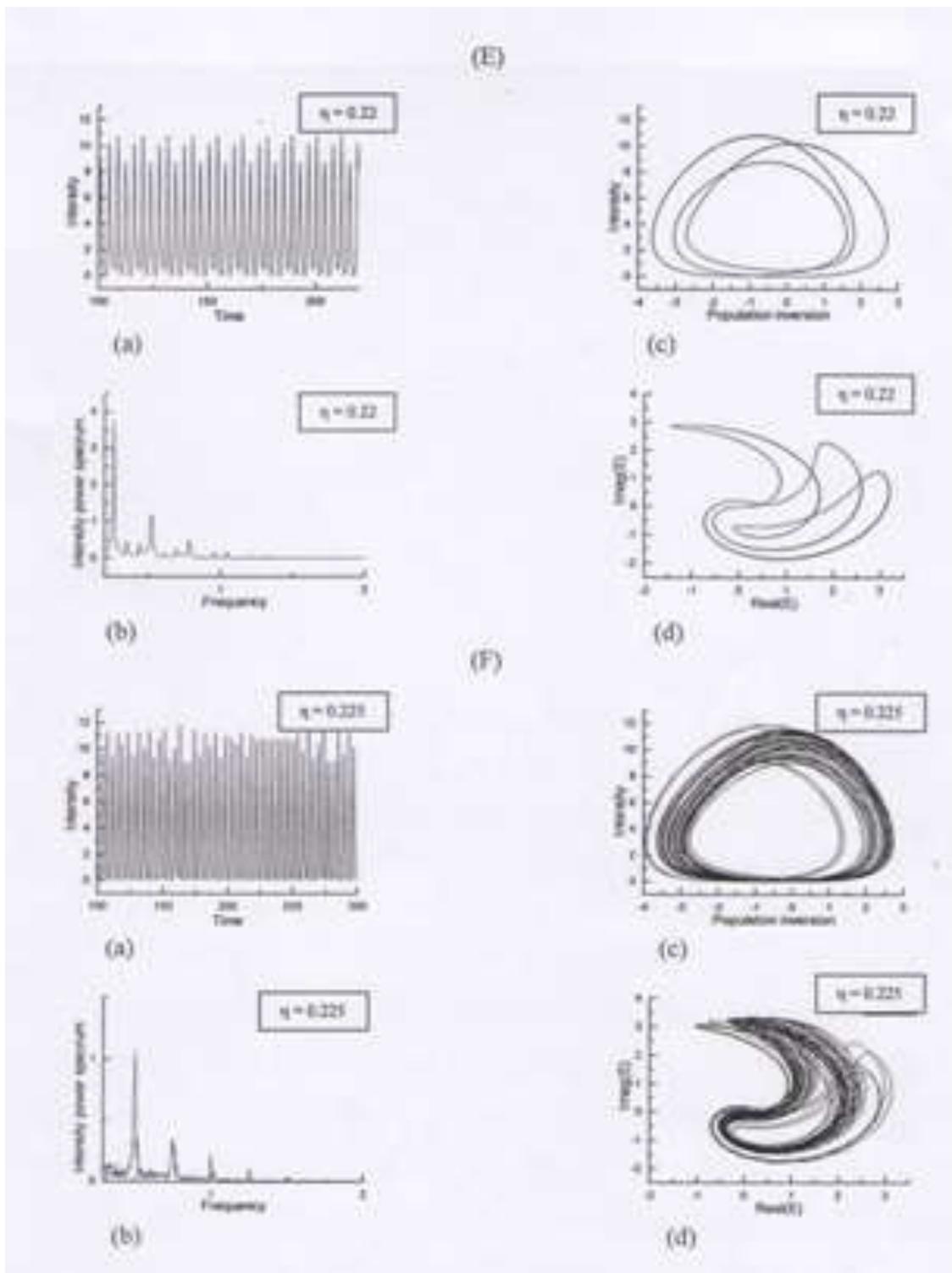


Fig.3

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الاستقرائية والعشوائية في ليزر أشباه الموصلات مع الحقن البصري والتغذية الارجاعية

فرات احمد عماد الدين

قسم الفيزياء – كلية التربية- جامعة البصرة- البصرة- العراق

الخلاصة

تم استخدام الطرق العددية لدراسة تأثيرات الحقن البصري والارجاع البصري على سلوك الديناميكية اللاخطية لليزر أشباه الموصلات باستخدام نموذج مبسط مبني على مزدوج المجال الكهربائي ومعادلات النسب للتعداد العكسي. مجموعة من الحالات المنفصلة تم ايضاحها . هذه تتضمن عملية الدورة المحدودة الدورية ، شبه الدورية والسلوك العشوائي يعتمد على نوع معامل السيطرة البصرية والشروط التشغيلية.