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Single – Photon Interference with Thermally Excited Three – Level Cascade Atoms

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Abstract

A theoretical model is presented for a single – photon interference experiment in a beam - splitter using the second photon in a cascade transition of a three – level atomic system. A clear anticorrelation effect is found in the coincidences of photons on the two sides of the beam splitter that can only be explained by quantum theory of light . A qualitative comparison is made with an earlier single – photon interference experiment available in literature.

Key wards: Single – Photon interference, Quantum Optics.

1.Introduction

The subject of photon – interference has received a continuous attention during the last two decades or so (for a review, see Mandel **(1999)**) .This interest has two main reasons. First, the photon - interference effect is a matter of contradiction between the classical and quantum theories of light, and thus it can throw some light on fundamentaltheoretical issues (Loudon **(1990)**). Second , the effect can also be used to prepare and study special quantum optical states that can be of great importance in important practical applications such as quantum cryptography , quantum teleportation (Kulik, et al. **(2004)**) .

The first experimental evidence of photon interference came in a series of experiments by Mandel and coworkers (see , for example , Hong , et , al. **(1987)**). Eversince , a large volume of theoretical and experimental work has followed (Legero,et.al **(2003)**). This volume of work has mostly relied on the system of parametric down conversion as a source of photons. Fearn and Loudon **(1989)** have given a theoretical model for an experiment that uses a thermally -driven three- level atomic system as a source for pairs of correlated photons in a two - photon interference experiment on a beam – splitter . Al – Gaim **(1998)** has extended the range of the experimental parameters in this model. Al - Khoza'i **(2005)** has formulated a theory for the same experiment using three - level cascade atomic system that is coherently driven by two radiation fields .

Single –photon interference can be a more radical effect than that of two –photon interference. According to the quantum theory of light ,a single photon can be either reflected or transmitted on a beam – slitter , and thus can only be registered once by detectors on both sides of the beam – splitter . Anticorrelation effect is thus expected in any experiment that registers the rate of photon coincidences on both sides of the beam - splitter. In this sense, Grangier et. al. **(1986)** had experimentally registered a strong anticorrelation between the triggered detection on both sides of a beam – splitter using an atomic cascade as a light source in a triggered detection scheme for the second photon of the cascade .

The aim of the present work is to formulate a theory for a similar experiment that registers the rate of photon coincidences on both sides of a beam – splitter using the second photon of a thermally excited three – level cascade atomic system . The next section contains the formulation of the problem, while section **(3)** shows a discussion of the results.

2. Theortical Model

Figure **(1)** represents a schematic diagram of the settings of a single – photon interference experiment with a beam – splitter **(BS)** , a source of photons **(S)** and two detectors ($\mathbf{D}_2$ and $\mathbf{D}_3$) in the two out put arms of the **(BS)** . The source of photons is a three – level cascade atomic system (Fig . (2)) . A triggered detection scheme is used in which the first photon in the cascade emission is used as a trigger that opens a gate during which the rate of arrival of the second photon is registered in and between the two detectors .The density matrix ρ of the atomic system in Fig. (2) is subject to the following rate – equations (Loudon (1980)) ;

$$\begin{aligned}
\frac{d}{dt}\rho_{11} &= 2\gamma_1\rho_{11} - R \\
\frac{d}{dt}\rho_{22} &= 2\gamma_2\rho_{33} - 2\gamma_1\rho_{22} \\
\frac{d}{dt}\rho_{33} &= R - 2\gamma_2\rho_{33} \\
\frac{d}{dt}\rho_{21} &= -\gamma_1\rho_{21} \;, \quad \frac{d}{dt}\rho_{32} = -(\gamma_1+\gamma_2)\rho_{32}
\end{aligned} \qquad \ldots(1)$$

Where R and γ_i are defined as in Fig .(2) .

During a total detection time T , the mean number of photons registered by detector (i) and mean number of photon coincidences between the two detectors (i and j) are , respectively , given by (Fearn and Loudon (1989)) ;

$$\begin{aligned}
\langle m_i \rangle &= \eta_i \int_0^T \Gamma^{(1)}(\vec{r}_i t;\vec{r}_i t)\, dt \\
\langle m_i m_j \rangle &= \eta_i \eta_j \int_0^T dt \int_0^T dt'\, \Gamma^{(2)}(\vec{r}_i t \;,\; \vec{r}_j t' ;\; \vec{r}_j t' \;,\; \vec{r}_i t)
\end{aligned} \qquad \ldots(2)$$

where the functions in the integrands are, respectively , the first and second – order coherence of the detected light field given by :

$$\begin{aligned}
\Gamma^{(1)}(\vec{r}_i t \;;\; \vec{r}_i t) &= \left\langle \hat{E}^-(\vec{r}_i t)\hat{E}^+(\vec{r}_i t) \right\rangle \\
\Gamma^{(2)}(\vec{r}_i t, \vec{r}_j t' ; \vec{r}_j t', \vec{r}_i t) &= \left\langle \hat{E}^-(\vec{r}_i t)\hat{E}^-(\vec{r}_j t')\,\hat{T}\,\hat{E}^+(\vec{r}_j t')\hat{E}^+(\vec{r}_i t) \right\rangle
\end{aligned} \qquad \ldots(3)$$

In (2) η_i is a parameter that contains the efficiency of detector (i) . In (3) , $\hat{T}$ is a time - ordering operator and $\hat{E}^+$ and $\hat{E}^-$ are, respectively the annihilation and creation operators of the detected optical field .These operators can easily be related to the atomic transition operators (Loudon (1983)). The resulting expectation values of the atomic transition operators can be expressed in terms of the matrix elements of the atomic density operator ρ . Multi – time expectation values of the atomic transition operators in the degree of second – order coherence can be evaluated using the quantum regressing theory (Lax **(1968)**) . Expressing the solutions of **(1)** in the form

$$\rho_{ij}(t') = \sum_{kl} \alpha_{kl}^{(ij)}(t'-t)\rho_{kl}(t) \qquad , t' > t \qquad \ldots(4)$$

Where α_{kl} is a scalar quantity that depends as $(t'-t)$, and employing the above outlined procedure (see, for example Al – Gaim **(1998)**) , one can obtain the fallowing expressions ;

$$\Gamma^{(1)}(\vec{r}_2 t\,,\ \vec{r}_2 t) = \left| \mathscr{r} \right|^2 E_o^2(\vec{r}_1)\rho_{22}(t - \frac{r_1}{c})$$

$$\Gamma^{(1)}(\vec{r}_3 t\,,\ \vec{r}_3 t) = \left| \mathscr{t} \right|^2 E_o^2(\vec{r}_1)\rho_{22}(t - \frac{r_1}{c})$$

$$\Gamma^{(2)}(\vec{r}_2 t, \vec{r}_3 t'; \vec{r}_3 t', \vec{r}_2 t) = \left| \mathscr{r}\,\mathscr{t} \right|^2 E_o^4(\vec{r}_1)\alpha_{11}^{(22)}(t'-t)\,\rho_{22}(t - \frac{r_1}{c}) \qquad , (t' > t)$$

$$= \left| \mathscr{r}\,\mathscr{t} \right|^2 E_o^4(\vec{r}_1)\alpha_{11}^{(22)}(t-t')\,\rho_{22}(t' - \frac{r_1}{c}) \qquad , (t > t') \qquad \ldots(5)$$

In this equation $E_o(\vec{r})$ is a scalar function that describes the spatial distribution of the field, $\mathscr{r}$ and $\mathscr{t}$ are respectively the reflection and transmition coefficients of the beam – splitter.

3. Results

Employing the solutions of **(1)** into **(5)** and performing the integrals in **(2)** taking into account time – ordering indicated in **(3)** , one can finally get the following results for steady – state conditions

$$\langle m_2 \rangle = \eta_2 \left| \mathscr{r} \right|^2 E_o^2(\vec{r}_1)\rho_{22}(\infty)T$$

$$\langle m_3 \rangle = \eta_3 \left| \mathscr{t} \right|^2 E_o^2(\vec{r}_1)\rho_{22}(\infty)T$$

$$\langle m_2 m_3 \rangle = \eta_3\eta_2 \left| \mathscr{r}\,\mathscr{t} \right|^2 E_o^4(\vec{r}_1)\rho_{22}(\infty)\left[\frac{RT^2}{2\gamma_1} + \frac{\gamma_2 R}{2\gamma_1^2(\gamma_1-\gamma_2)}\left\{ T + \frac{1}{2\gamma_1}(e^{-2\gamma_1 T} - 1) \right\} \right.$$

$$\left. - \frac{R}{2\gamma_2(\gamma_1-\gamma_2)}\left\{ T + \frac{1}{2\gamma_2}(e^{-2\gamma_2 T} - 1) \right\} \right] \qquad \ldots(6)$$

Let us define an "Anti – correlation parameter" α by

$$\alpha = \frac{\langle m_2 m_3 \rangle}{\langle m_2 \rangle \langle m_3 \rangle} \qquad \ldots(7)$$

Define the following normalized quantities,

$$r = \frac{R}{\gamma_1} \quad , \quad \gamma = \frac{\gamma_2}{\gamma_1} \quad , \qquad X = RT \qquad \ldots(8)$$

Then, α can be expressed as

$$\alpha = 1 + \frac{1}{2\gamma^2 X^2}\left[r^2 + \gamma r(1+\gamma)(r-2X) + \frac{r^2}{(1-\gamma)}\left\{ \gamma^3 e^{-\frac{2X}{r}} - e^{-\frac{2\gamma X}{r}} \right\} \right] \qquad \ldots(9)$$

According to the classical wave – description of the experiment, the number of photon counts indicated above would correspond to ensemble averages of intensities impinging on the beam splitter. Cauchy – Schwarz inequality would hold for such intensity averages (Loudon **(1983)**), which means that

$$\langle m_2 m_3 \rangle \geq \langle m_2 \rangle \langle m_3 \rangle \quad , \qquad \ldots(10)$$

giving

$$\alpha \geq 1 \qquad \ldots(11)$$

The violation of inequality **(11)** thus gives "anti – correlation" criterion for characterizing a nonclassical behavior (Grangier et . al . **(1986)**) .

Fig . **(3)** shows four examples of the parameter α given in **(9)** . Inequality **(11)** is always violated in a clear demonstration of single – photon interference effect on a beam – splitter. For experimental conditions in which $R > \gamma_1$ and $\gamma_2 < \gamma_1$, the effect is deeply demonstrated . In Fig . **(3b)** , we find single – photon interference with visibility that exceeds that registered by (Grangier et. al. **(1986)**) by several orders. In their experimental work , (Grangier et . al **(1986)**) have given no information on the data of their atomic system a part form indicating that it was a three – level cascade atomic system . Thus, we can not make any quantitative comparison with their experimental graphs. However, we note an excellent qualitative agreement between our predicted graphs and their mentioned experimental graph.

In conclusion, we have demonstrated the ability of thermally – excited three – level cascade atomic system to act as a source for "single – photon states".

Figure Captions

1- Fig. (1) ; A Schematic diagram of the experimental setting of a single – photon interference on a beam – splitter (BS), showing the source (S) of the single – photon states and two detectors (D_2 and D_3) on both sites of the (BS).

2- Fig. (2) ; A Schematic diagram of thermally excited three – level cascade atomic system, showing the different rates of transition.

3- Fig. (3) ; Four examples of the anti – correlation parameter (α) as a function of $(x = RT)$.

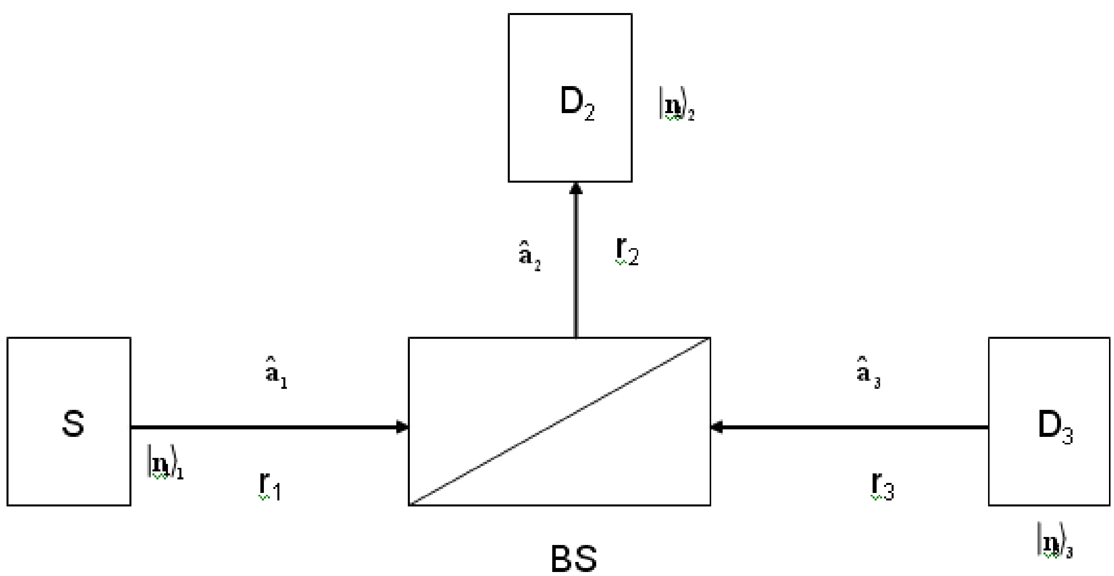

Fig . (1)

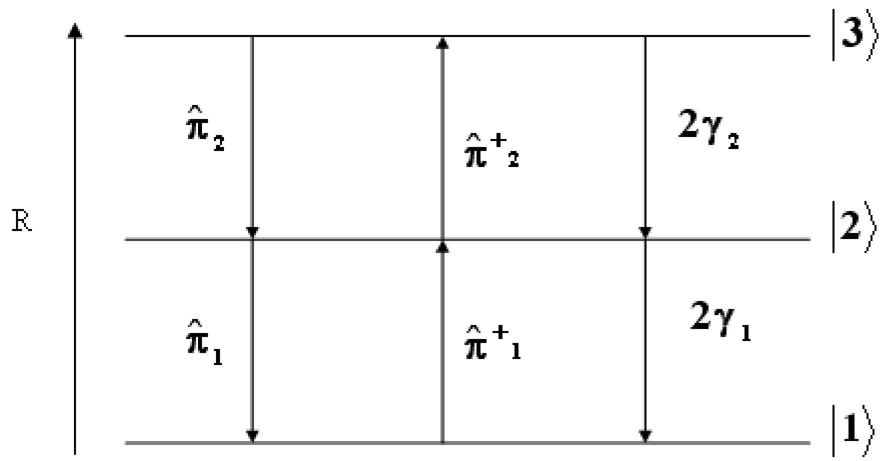
|3⟩
|2⟩
|1⟩
R
$\hat{\pi}_2$
$\hat{\pi}^+_2$
$2\gamma_2$
$\hat{\pi}_1$
$\hat{\pi}^+_1$
$2\gamma_1$

Fig. (2)

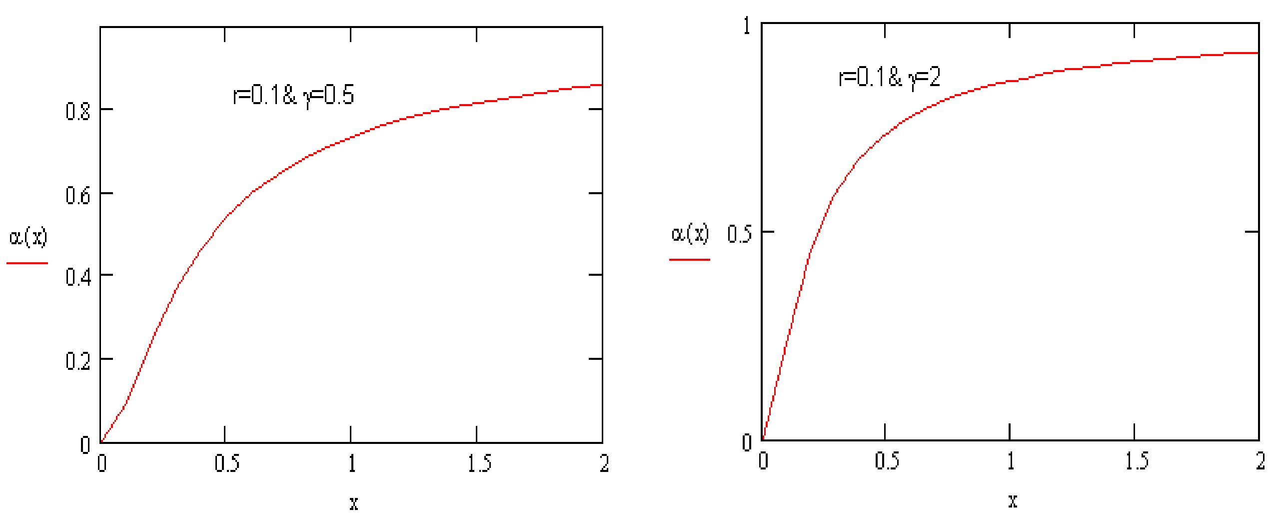
r=0.1& γ=0.5
α(x)
x
r=0.1& γ=2
α(x)
x

Fig. (3a)

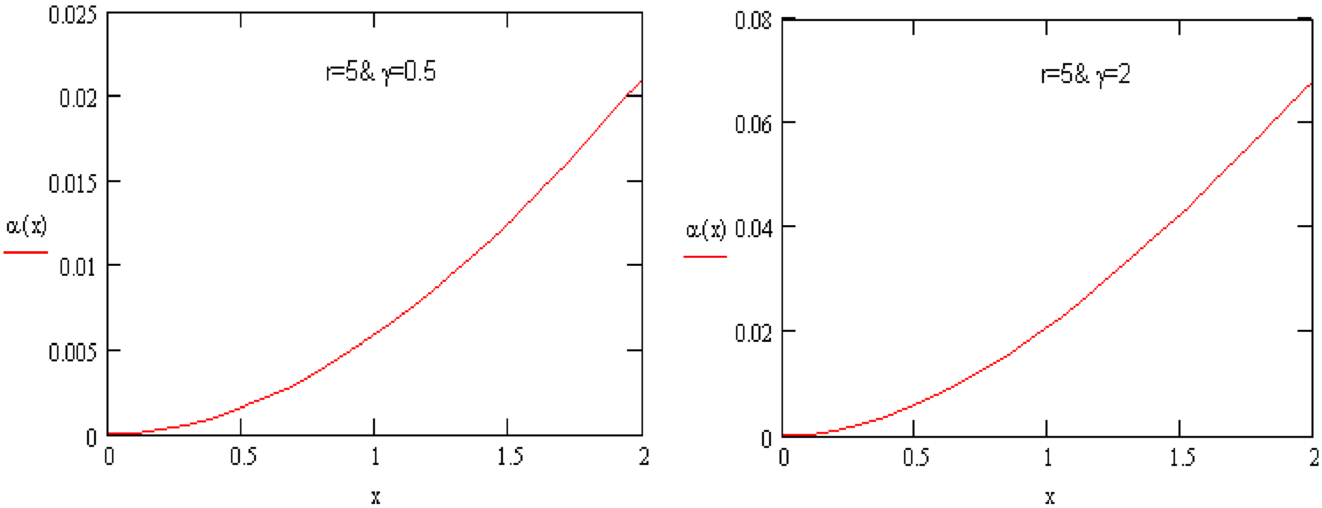

Fig. (3b)

References

1. Al - Gaim,H.R.M., (1998), M.Sc. Thesis, College of Education , University of Basrah, Iraq.

2. Al-Khoza'i,T.A.,(2005) ,M.Sc. Thesis, College of Education ,University of Basrah, Iraq.

3. Fearn, H. and Loudon , R .(1989) , J.opt.Soc.Am.,B6,917 .

4. Gragier,P,Roger,G.and Aspect, A.(1986) , Europhys.Lett.1,173.

5. Hong,C.K.,Ou,Z.Y.and Mandel,L., (1987),Phy.Rev.Lett.59, 2044.

6. Kulik, S.P.,Maslennikov, G.A.,Merkulova ,S.P.,Penin,A.N., Radchenko,L.K.,andKrasheninnikov ,V. N., (2004), J.Exp . and Theor .Physics,98,31.

7. Lax , M.(1968),Phys.Rev.172,350.

8. Legero, T.,Wilk,T. Kuhn, A. and Rempe , G. (2003), Appl.Phys.B77,797.

9. Loudon,R.(1980),Rep.Prog.Phys.43,913

10. Loudon,R.(1983),"The quantum theory of light" , 2nd edn .(Oxford ;Clarendon).

11. Loudon , R. (1990) in " Disorders in Condensed Matter Physics " , Ed . by Blackman and Tayuena , Clarendon Press ; Oxtord .

12. Mandel, L. (1999) , Reviews of Mod . Phys. 17 , S274 .

تداخل – الفوتون الواحد بأستعمال ذرات ثلاثية المستوي متعاقبة الانبعاث مهيجة حرارياً

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الملخص

نقدم نموذجاً نظرياً لتجربة تداخل – الفوتون الواحد بأستعمال مجزئ الشعاع الضوئي وتوظيف الفوتون الثاني في الانبعاث المتعاقب لنظام ذري ثلاثي المستوي.تم الحصول على تأثيرات واضحة لعكس – الارتباط في معدلات التوافق للفوتونات على طرفي مجزئ الشعاع الضوئي. مثل هذه التأثيرات يمكن فقط تفسيرها بالاعتماد على النظرية الكمية للضوء . تمت مقارنة النتائج نوعياً مع تجربة تداخل – الفوتون الواحد متوفرة في أدبيات الموضوع .