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Studying the Synchronization for Multiple Transmitter- Receiver Nano Quantum Cascade Lasers.

Hiba Bahjat*[1a] and Hussein H. Waried [1b]

[1] *Physics Department, College of Science, University of Thi-Qar, Thi-Qar, Iraq*

[b]E-mail: hussein.hadi@sci.utq.edu.iq
[a*]Corresponding author: hiba.reasn@utq.edu.iq

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***Abstract*— In this paper, the chaos synchronization of four Nano quantum cascade lasers with optoelectronic feedback was investigated. The present system consists of two receiver lasers and two transmitters lasers. The rate equation model was modulated to study the effect of the Purcell factor F and the spontaneous emission factor because of their importance in a nanocavity. The results indicated that the suggested system, realize the chaos synchronization. Also, the nanocavity parameters F and β, have significant effect on the chaos synchronization quality and may lead to lose it. Furthermore, the results showed the weak effect of delay time on the performance of lasers.**

***Keywords*— Nano quantum cascade laser (NQCLs), transmitter laser, receiver laser, synchronization.**

I. INTRODUCTION

In recent years, the rapid advancement of nanotechnology and quantum mechanics has revolutionized the field of photonics, leading to the development of an array of innovative optical devices. One of the groundbreaking invention is the Nano Quantum Cascade Laser (NQCL). NQCL employs quantum cascade effect within nanoscale structures to emit coherent light in the mid-infrared spectrum [1-5]. Nano QCLs are semiconductor-based devices that utilize transitions between quantized energy states in ultra-thin layers to achieve photon emission [6-10]. A key advantage of Nano QCLs is their abilities to operate in the mid-infrared region, where numerous chemical compounds exhibit unique absorption characteristics. This feature enables a wide range of applications, including gas sensing, explosives detection, environmental monitoring, and biomedical diagnostics. The operation of a Quantum Cascade Laser based on the concept of barrier height engineering [11-14]. Multiple quantum wells are stacked together in a cascaded structure, forming a potential energy ladder. Excitation of electrons from the ground state to higher energy states occurs within the active layer, and as electrons transit between quantum wells, they emit photons due to resonant tunneling [15,16] Nano QCLs have revolutionized several fields, such as gas sensing, medical diagnostics, laser-based manufacturing, and communication. As the understanding of nanoscale physics and quantum mechanics continues to develop, further advancements in Nano QCLs are anticipated. With ongoing research efforts, it is expected that Nano QCLs will find new applications, helping solve complex real-world problems and contributing to advancements in science and technology [17]. Optoelectronics refer to a branch of electronics that deals with Feedback, and it is a fundamental concept in electronics and plays a crucial role in improving the performance of electronic systems [18] [19]. Negative optoelectronics feedback refers to the use of feedback loops in optoelectronic devices to enhance their performances and stabilities [20][21]. Optoelectronics refers to the branch of electronics that deals with the control, generation, and detection of light, such as in lasers, light-emitting diodes (LEDs), and photodetectors. Negative optoelectronics feedback can also be employed to stabilize the output intensity of laser [22]. Optoelectronics feedback is a technique that was used to achieve synchronization by employing optical signals. It utilizes the properties of light, such as speed, reliability, and high bandwidth, to facilitate the transfer of information and coordination among components. Optoelectronics feedback systems consist of a light source, a photodetector, and an electronic feedback loop that detects and adjusts the phase or frequency of the optical signal. The feedback loop ensures that the system remains in a synchronized state by continuously correcting any deviation from the desired synchronization pattern [23.24].Chaos synchronization is an emerging field that deals with the study of synchronizing chaotic systems. Chaotic systems are nonlinear and inherently complex, displaying sensitive dependence on initial conditions. Over the years, researchers have made significant progress in understanding the principles of chaos synchronization, leading to various applications in numerous fields. One of the prominent and widely studied applications of chaos synchronization is in secure communication systems, Purcell factor which enhances or suppresses the spontaneous emission factor, by which the lifetime of photon emission is shortened [25].

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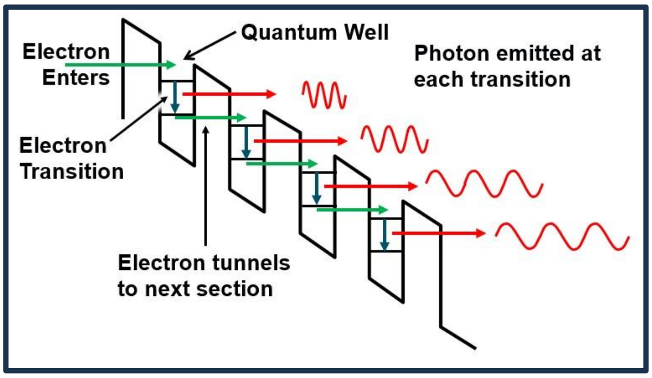

Fig. 1: Nano quantum cascade lasers design [9].

In this paper, a theoretical study about the synchronization of multiple transmitter-receiver nano quantum cascade lasers was presented for the first time to my knowledge. The motivation behind the present work was the need for a small, integrated, and efficient source with excellent dynamic and static properties in many emerging areas of processing photonic signals.

II. SYSTEM MODEL

The proposed system consists of a pair of transmitters and a pair of receivers. Each one consists of a cascade nanoscale quantum laser, a photodetector, and an amplifier. As showed in the coming equations and figures, part of the laser output from any transmitter transmitted to the laser itself as a feed. The other part collected by the part coming from the second transmitter laser and conveyed to the transmitting lasers equally. Thus, the resulted current added to the current of the laser device.

TNQCL 1, TNQCL 2: transmitters Nano quantum cascade lasers. $PD_{T1,2}$: photodetector for transmitters. $PD_{R1,2}$ photodetector for receivers. $A_{T1,2}$: amplifiers for transmitters. $A_{R1,2}$: amplifiers for receivers. can be stated using the rate equation that as follow.

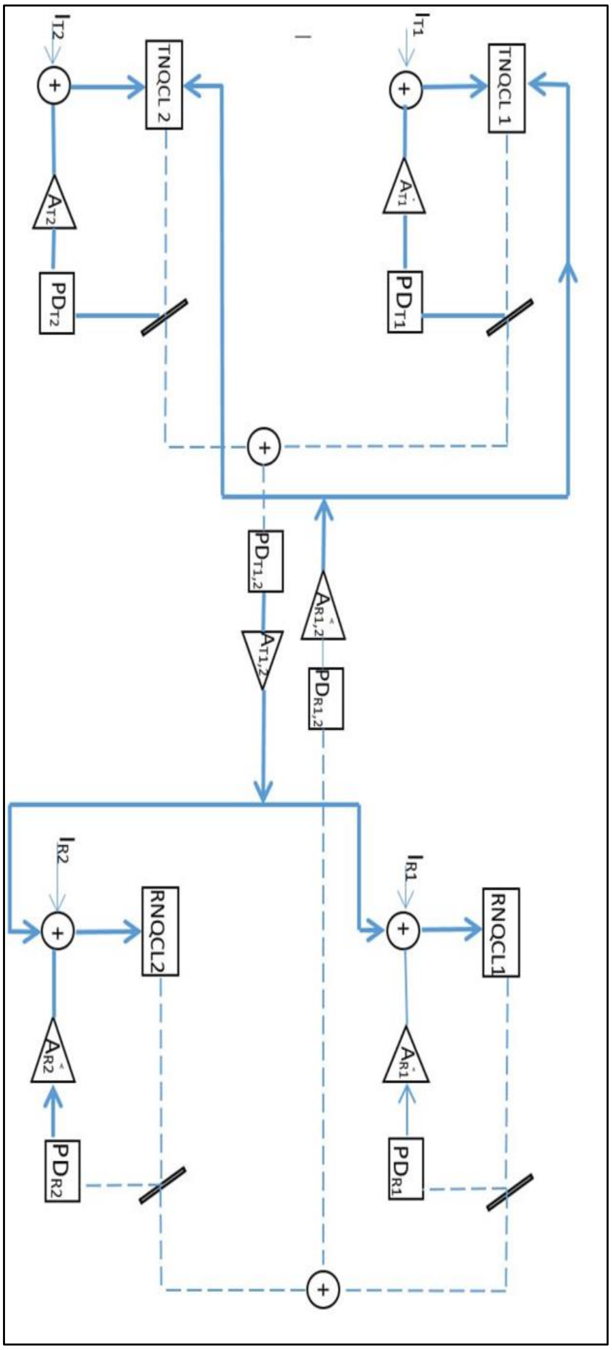

Fig. 2: Displays an illustration of a synchronization system schematic.

$$\frac{dS_{T1}}{dt} = NG(N_{3T1} - N_{2T1})S_{T1} - \frac{S_{T1}}{\tau_p} + (NGN_{1T1}S_{T1})\frac{\beta N_{3T1}}{\tau_{sp}}(S_{T1} + 1) + NGN_{3T1}S_{T1} \qquad (1)$$

$$\frac{dN_{3T1}}{dt} = \eta\frac{I}{q}\Big(1 + [\zeta_{T1}S_{T1}(t-\tau_{T1}) + \zeta_{cPR1}\,S_{R1}(t-\tau_C) + \zeta\, c_{PR2}\,S_{R2}(t-\tau_C)] - \frac{S_{T1}\beta N_{3T1}}{\tau_{sp}} - G(N_{3T1} - N_{2T1}\Big)S_{T1} - NGN_{3T1}S_{T1} \qquad (2)$$

$$\frac{dN_{2T1}}{dt} = \frac{N_{3T1}}{\tau_{32}} - \frac{N_{2T1}}{\tau_{21}} + G(N_{3T1} - N_{2T1})S_{T1} \qquad (3)$$

$$\frac{dN_{1T1}}{dt} = \frac{N_{3T1}}{\tau_{31}} - \frac{N_{2T1}}{\tau_{21}} - \frac{N_{1T1}}{\tau_{out}} \qquad (4)$$

$$\frac{dS_{R1}}{dt} = NG(N_{3R1} - N_{2R1})S_{R1} - \frac{S_{R1}}{\tau_p} + (NGN_{1R1}S_{R1})\frac{\beta N_{3R1}}{\tau_{sp}}(S_{R1} + 1) + NGN_{3R1}S_{R1} \qquad (5)$$

$$\frac{dN_{3R1}}{dt} = \eta \frac{I}{q}\Big(1 + \big[\zeta_{R1} S_{R1}(t - \tau_{R1}) + \zeta_{cPT1}\, S_{T1}(t - \tau_C) + \zeta\, c_{PT2}\, S_{T2}(t - \tau_C)\big] - \frac{S_{R1}\beta N_{3R_1}}{\tau_{sp}} - G(N_{3R1} - N_{2R1}\Big) S_{R1} - NGN_{3R1}S_{R1} \quad (6)$$

$$\frac{dN_{2R1}}{dt} = \frac{N_{3R1}}{\tau_{32}} - \frac{N_{2R1}}{\tau_{21}} + G(N_{3R1} - N_{2R1})S_{R1} \quad (7)$$

$$\frac{dN_{1R1}}{dt} = \frac{N_{3R1}}{\tau_{31}} - \frac{N_{2R1}}{\tau_{21}} - \frac{N_{1R1}}{\tau_{out}} \quad (8)$$

$$\frac{dS_{T2}}{dt} = NG(N_{3T2} - N_{2T2})S_{T2} - \frac{S_{T2}}{\tau_p} + (NGN_{3T2}S_{T2})\frac{\beta N_{3T2}}{\tau_{sp}}(S_{T2} + 1) + NGN_{3T2}S_{T2} \quad (9)$$

$$\frac{dN_{3T2}}{dt} = \eta \frac{I}{q}\Big(1 + \big[\zeta_{T2} S_{T2}(t - \tau_{T2}) + \zeta_{cPR1}\, S_{R1}(t - \tau_C) + \zeta\, c_{PR2}\, S_{R2}(t - \tau_C)\big] - \frac{S_{T2}\beta N_{3T_2}}{\tau_{sp}} - G(N_{3T2} - N_{2T2}\Big) S_{T2} - NGN_{3T2}S_{T2} \quad (10)$$

$$\frac{dN_{2T2}}{dt} = \frac{N_{3T2}}{\tau_{32}} - \frac{N_{2T2}}{\tau_{21}} + G(N_{3T2} - N_{2T2})S_{T2} \quad (11)$$

$$\frac{dN_{1T2}}{dt} = \frac{N_{3T2}}{\tau_{31}} - \frac{N_{T2}}{\tau_{21}}\frac{N_{T2}}{\tau\, out} \quad (12)$$

$$\frac{dS_{R2}}{dt} = NG(N_{3R2} - N_{2R2})S_{R2} - \frac{S_{R2}}{\tau p} + (NGN_{3R2}S_{R2})\frac{\beta N_{3R2}}{\tau_{sp}}(S_{R2} + 1) + NGN_{3R2}S_{R2} \quad (13)$$

$$\frac{dN_{3R2}}{dt} = \eta \frac{I}{q}\Big(1 + \big[\zeta_{R2} S_{R2}(t - \tau_{R2}) + \zeta_{cPT1}\, S_{T1}(t - \tau_C) + \zeta\, c_{PT2}\, S_{T2}(t - \tau_C)\big] - \frac{S_{R2}\beta N_{3R_2}}{\tau_{sp}} - G(N_{3R2} - N_{2R2}\Big) S_{R2} - NGN_{3R2}S_{R2} \quad (14)$$

$$\frac{dN_{2R2}}{dt} = \frac{N_{3R2}}{\tau_{32}} - \frac{N_{2R2}}{\tau_{21}} + G(N_{3R2} - N_{2R2})S_{R2} \quad (15)$$

$$\frac{dN_{1R2}}{dt} = \frac{N_{3R2}}{\tau_{31}} - \frac{N_{R2}}{\tau_{21}} - \frac{N_{R2}}{\tau_{out}} \quad (16)$$

Here, the indices $T_{1,2}$ represent the first and second transmitters, $R_{1,2}$ the first and second receivers and ,N_3, N_2, and N_1 are the carrier numbers in levels 3, 2, and 1, respectively. q is the electron's charge. The injection current is Iin, and the injection rate is η. In laser circuits, $\tau_{T1,2,R1,2}$ is the feedback delay time, τ_{MP1}, τ_{CP2} are the transmission times between TNQCL1, TNQCL2 and RNQCL1, RNQCL2. The photon lifetime is τp, the spontaneous emission factor is β, the gain coefficient is G, and the phonon scattering times between levels are τ_{31}, τ_{32}, and τ_{21}. The carriers tunneling time is τ_{out}. Z represents the gain stages number; ζ_{T1}, ζ_{T2} are the feedback to quantitatively assess the synchronization quality between any transmitter and receiver laser, we utilize a correlation coefficient denoted as ρ.

$$\rho 1 = \frac{\langle [S_{T1}(t) - \langle S_{T1}(t)\rangle]\, [S_R(t) - \langle S_{R1}(t)\rangle]\rangle}{\langle |S_{T1}(t) - \langle S_{T1}(t)\rangle|^2\rangle^{\frac{1}{2}} \langle |S_R(t) - \langle S_{R1}(t)\rangle|^2\rangle^{\frac{1}{2}}} \quad (17)$$

$$\rho 2 = \frac{\langle [S_{T2}(t) - \langle S_{T2}(t)\rangle]\, [S_R(t) - \langle S_{R1}(t)\rangle]\rangle}{\langle |S_{T2}(t) - \langle S_{T2}(t)\rangle|^2\rangle^{\frac{1}{2}} \langle |S_R(t) - \langle S_{R1}(t)\rangle|^2\rangle^{\frac{1}{2}}} \quad (18)$$

$$\rho 3 = \frac{\langle [S_{R1}(t) - \langle S_{R1}(t)\rangle]\, [S_{T1}(t) - \langle S_{T1}(t)\rangle]\rangle}{\langle |S_{R1}(t) - \langle S_{R1}(t)\rangle|^2\rangle^{\frac{1}{2}} \langle |S_{T1}(t) - \langle S_{T1}(t)\rangle|^2\rangle^{\frac{1}{2}}} \quad (19)$$

$$\rho 4 = \frac{\langle [S_{R2}(t) - \langle S_{R2}(t)\rangle]\, [S_{T2}(t) - \langle S_{T2}(t)\rangle]\rangle}{\langle |S_{R2}(t) - \langle S_{R2}(t)\rangle|^2\rangle^{\frac{1}{2}} \langle |S_{T2}(t) - \langle S_{T2}(t)\rangle|^2\rangle^{\frac{1}{2}}} \quad (20)$$

III. RESULTS AND DISCUSSION

The primary goal of this paper was to examine how well synchronization workd in nano quantum cascade lasers, with a special emphasis on the roles t of the Purcell factor F and the spontaneous emission coupling factor β play. The dynamics of the nano lasers are examined for the device parameters in Table 1 and the results presented here were assessed using the rate equations (1)– (20).

TABLE 1. PARAMETER VALUES USED FOR SYNCHRONIZATION CALCULATIONS [10].

Symbol	*Value*	*Unit*
η	0.4	
β	0.99	
N	20	
F	10	
I	0.5	mA
G	1.2×10^{5}	s^{-1}
τ_{sp0}	1×10^{-9}	Ns
τ_p	0.26×10^{-12}	Ps
τ_{32}	0.26×10^{-12}	Ps
τ_{31}	0.26×10^{-12}	Ps
τ_{21}	0.5×10^{-12}	Ps
τ_{out}	0.54×10^{-12}	Ps

Fig. 3. shows the results of present simulation according to solve the theoretical model that suggested in an open-loop optoelectronic feedback circuit. The chaotic temporal waveforms, phase portrait, and correlation plot between the RNQCL and TNQCL waveforms with ρ=1 were presented.

Under match conditions between all lasers, fully coupled two lasers produced extremely complex chaotic outputs. Similar variation properties between TNQCL and RNQCL were noticed, indicating that the system can realize synchronization under the previously chosen values. The low values of bias injection current and photon lifetime, respectively, are the causes of the low carrier number and photon number. From the results in the figure, during the transmission of secure encrypted messages in other types of lasers, the quality of the received messages was related to the quality of synchronization between the lasers. From the results in the figure, we can clearly see that the proposed system is perfectly suitable for transmitting secure encrypted messages between sending and receiving lasers. According to the results, secure messages can be transmitted between any pair of sending-receiving lasers due to their complete synchronization. Eqs. 17-20 were used to calculate the quality of synchronization between the transmitting and receiving lasers. The quality of synchronization was through the diagonal line and at the angle 45 between the transmitter photons and the receiver photons.

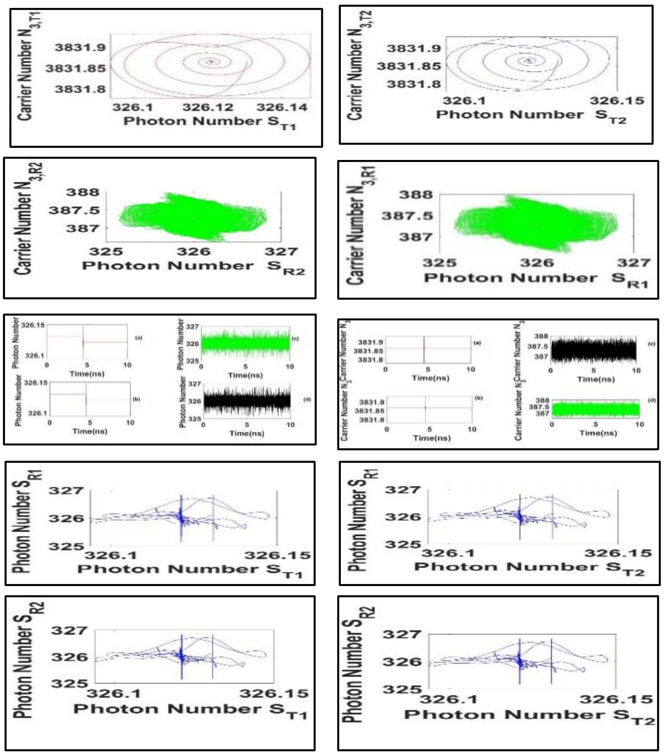

Fig.3: Phase portrait, chaotic temporal waveforms, and the correlation plot between the waveforms of TNQCL1, TNQCL2 and RNQCL ,RNQCL 2 with, $\tau_T = \tau_R = \tau_C = 4.5ns$ and $\rho_1=\rho_2=\rho_3=1$

Fig. 4. represents the effect of the spontaneous emission factor on chaotic synchronization when its value is different between the receiver laser and the transmitter laser. The difference in the value of the factor means the difference in the amount of contribution of spontaneous emission to the laser output and thus a difference in the number of photons generated in each laser. Because synchronization is very sensitive to the initial conditions, we noticed a complete difference in the performance of each laser. Two important things must be pointed out here. First, during this study we did not present a formula to describe the relationship between Purcell factor to the spontaneous emission factor because the relationships are multiple, and their studies are still ongoing due to the novelty of the topic. The other thing is that a large value of the spontaneous emission coefficient reduces the number of carriers, as shown in the figures, unlike the case in lasers with a small value.

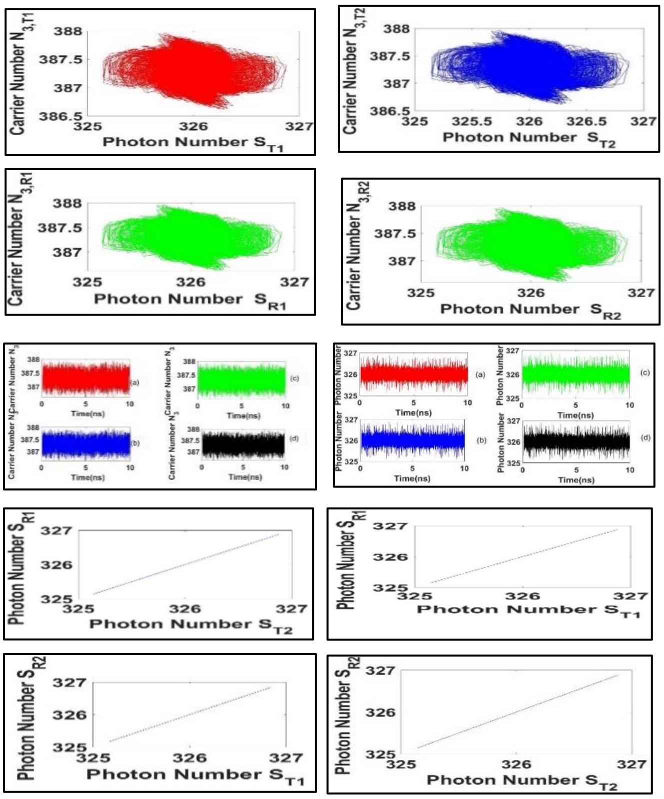

Fig.4. Phase portrait, chaotic temporal waveforms, and the correlation plot between the waveforms of TNQCL1, TNQCL2 and RNQCL, RNQCL2 with, $\tau_T = \tau_R = \tau_C = 4.5ns$ and $\rho_1=\rho_2=$, $\rho_3=\rho_4=$ 0.0241

Fig. 5. represents the effect of the Purcell factor on chaotic synchronization when its value is equal to 10 in all transmitter lasers and 40 in the all receiver lasers. The difference in the value of the factor lead to a difference in the spontaneous emission time in the nan cavity and thus a difference in the generated instantaneous photons that contribute to the laser output. When calculating the coupling coefficient, we found it equal to 0.0332. Plotting the photons of the transmitting lasers with the receiver, we clearly found that the synchronization disappeared because the synchronization process was very sensitive to the initial conditions of the factors presenting in the rate equations model.

From Table 2, we can see that the weak effect of the feedback coefficients and delay times on the chaos synchronization. These results were noted in several previous works. Also, Table 3. shows that the effect of delay times values on the chaos synchronization. The Purcell factor, which affects the spontaneous emission rate, can impact the synchronization behavior of quantum systems in the presence of delay times. A high Purcell factor can mitigate the negative effects of delay times by enhancing the probability of synchronized photon emissions, thereby improving synchronization quality.

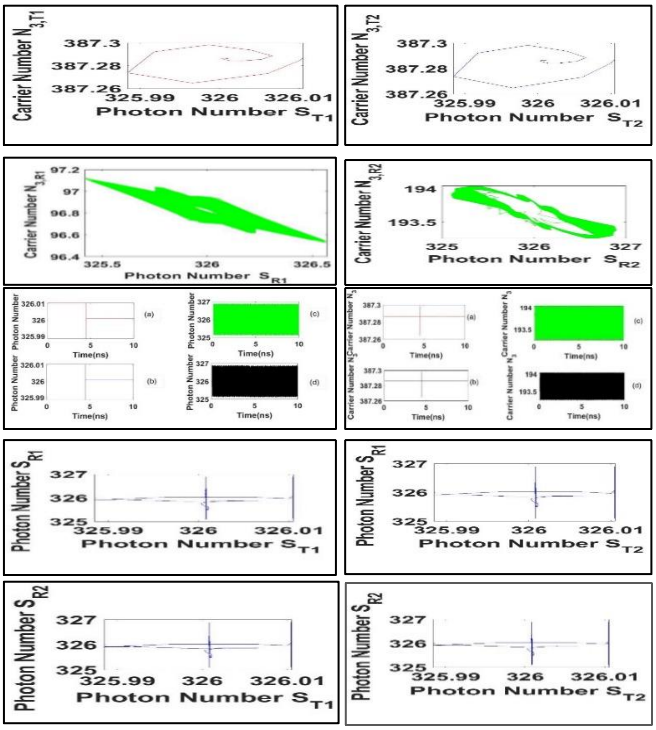

Fig. 5. Phase portrait, chaotic temporal waveforms, and the correlation plot between the waveforms of TNQCL1, TNQCL2 and RNQCL1, RNQCL2 with $\tau_T = 6 = \tau_R = \tau_C = 4.5ns$ and ρ 1= ρ2=, ρ3= ρ4=0.0332

TABLE 2. CHANGHANGE THE CORRELATION COFFICIENT AND ITS EFFECT ON CHAOS SYNCHRONIZTION [15].

	ζ_T	ζ_R	ζ_{C1}	ζ_{C2}	$\rho 1$	$\rho 2$	$\rho 3$	$\rho 4$
1	α	α	α	α	1	1	1	1
2	β	α	α	α	0.9960	0.9960	0.9960	0.9960
3	α	β	α	α	0.9971	0.9971	0.9971	0.9971
4	α	α	β	α	1	1	1	1
5	α	α	α	β	1	1	1	1
6	β	β	α	α	1	1	1	1
7	α	α	β	β	1	1	1	1
8	β	α	β	α	0.9946	0.9946	0.9946	0.9946
9	α	β	β	α	0.9945	0.9945	0.9945	0.9945
10	α	β	α	β	0.9945	0.9945	1	0.9945
11	α	β	γ	σ	0.9982	0.9982	0.9982	0.9982

TABLE 3. EFFECT OF DELAY TIMES VALUES ON THE CHAOS SYNCHRONIZTION [15].

τ_T	τ_R	τ_{C2}	ρ_1	ρ_2	ρ_3	ρ_4
1.5	3	4.5	0.9789	0.9789	0.9789	0.9789
3	3	4.5	1	1	1	1
3	4.5	3	0.9901	0.9901	0.9901	0.9901
4.5	3	3	0.9994	0.9994	0.9994	0.9994
4.5	4.5	4.5	1	1	1	1

IIII. CONCLUSIONS

With the use of four Nano quantum cascade lasers with negative optoelectronic feedback, chaos synchronization is theoretically investigated. The purpose of this study was to examine the quality of chaos synchronization and the effects of the Purcell factor, spontaneous emission factor, feedback coefficients, and coupling coefficients using a full rate equations model with a Purcell factor. The synchronization quality found to be significantly impacted by the Purcell factor and spontaneous emission factor, but was affected weakly by the delay times. Since the synchronization is sensitive to the initial conditions, the correlation coefficient can also be increased when the Purcell factor and spontaneous emission factor are the same in all lasers.

CONFLICT OF INTEREST

Authors declare that they have no conflict of interest.

REFERENCES

[1] J. Faist, F. Capasso, D. L. Sivco, C. Sirtori, A. L. Hutchinson, and A. Y. Cho, “Quantum cascade laser,” *Science (80).*, vol. 264, no. 5158, pp. 553–556, 1994.

[2] H. Cai, R. Liu, W. Zhang, and B. Yan ,”Quantum cascade lasers and their applications in trace gas sensing”, *Sensors*, 10(3),2331-2365,2010.

[3] H. Moser, D. Hofstetter, M. Beck, T. Aellen, and J. Faist, “Quantum cascade laser-based standoff detection of explosives and explosive precursors”, *Chemical reviews*, 109(3),PP.1618-1687,2009.

[4] R. Bhattacharyya, M. Danielewicz, P. Grzywacz, and F. Baumgartner, “ Quantum cascade lasers for infrared breath analysis. *Applied spectroscopy reviews*”, 52(9-10), 754-771, 2017.

[5] T. Grange, T. Greszik, D. Gubernatis, M. Hemingway, M. Amanti, I., Rochais, and D. Hofstetter, “Quantum cascade lasers and their applications: device structures, optical properties, and fabrication procedures”, *In ECOC* (p. Tu 3.5. 3), 2015.

[6] H. H. Waried, “Synchronization of quantum cascade lasers with mutual optoelectronic coupling,” *Chinese J. Phys*., vol. 56, no. 3, pp. 1113–1120, 2018.

[7] H. H. Waried, “Controlling the Synchronization of Quantum Cascade Lasers with Negative Optoelectronic Feedback by Direct Current Modulation,” *Iran. J. Sci. Technol. Trans. A Sci.*, vol. 43, no. 4, pp. 2001–2011, 2019.

[8] H. Waried, “Chaos synchronization of coupled nano-quantum cascade lasers with negative optoelectronic feedback,” *Eur. Phys. J. D*, vol. 73, no. 2, pp. 1–9, 2019.

[9] R. Sharma, H. Kaur, & M. Singh, "Recent Advances of Efficient Design of Terahertz Quantum-Cascade Lasers". *Plasmonic* 16, 449–461, 2021.

[10] A. G. Mohammed and H. H. Waried, “Investigation the dynamic of nano quantum cascade lasers with optoelectronics feedback,” *Samarra J. Pure Appl. Sci*., vol. 2, no. 2, pp. 48–56, 2021.

[11] M. C. Chiang, H.-F. Chen, and J.-M. Liu, “Experimental synchronization of mutually coupled semiconductor lasers with optoelectronic feedback,” *IEEE J. Quantum Electron*., vol. 41, no. 11, pp. 1333–1340, 2005.

[12] T. Deng, G.-Q. Xia, L.-P. Cao, J.-G. Chen, X.-D. Lin, and Z.-M. Wu, “Bidirectional chaos synchronization and communication in semiconductor lasers with optoelectronic feedback,” *Opt. Commun.*, vol. 282, no. 11, pp. 2243–2249, 2009.

[13] Q. Gu et al., “Purcell effect in sub-wavelength semiconductor lasers,” *Opt. Express*, vol. 21, no. 13, pp. 15603–15617, 2013

[14] J. S. T. Smalley, Q. Gu, and Y. Fainman, “Temperature dependence of the spontaneous emission factor in subwavelength semiconductor lasers,” *IEEE J. Quantum Electron.*, vol. 50, no. 3, pp. 175–185, 2014.

[15] J. Ohtsubo, “Theory of optical feedback in semiconductor lasers,” in *Semiconductor Lasers,* Springer, pp. 75–101,2013.

[16] D. M. Kane and K. A. Shore, “Unlocking dynamical diversity: optical feedback effects on semiconductor lasers,” *John Wiley & Sons*, 2005.

[17] N. Yu, L. Diehl, F. Capasso, A. Belyanin and A.K. Wo, "Self-Synchronization of Laser Modes and Multistability in Quantum Cascade Lasers", *Physical review letters* **13390**. pp1–4, 2011. doi:10.1103/PhysRevLett.106.133902.

[18] H. Waried, “Synchronization of quantum cascade lasers with negative optoelectronic feedback,” *Recent Adv. Electr. Electron. Eng*, vol. 11, no. 2, pp. 167–175, 2018.

[19] J.M.L. S. Tang and H.F. Chen, "Stable route-tracking synchronization between two chaotically pulsing semiconductor lasers", *Opt. Lett*. **26**.PP1489–91, 2001. doi:10.1364/OL.26.001489.

[20] B.J. Adamson and M.T. Westbrook, "Crafty Birds and Fitness Fiends: Relationships of Professional and Leisure Activities", *Aust. J. Physiother*. **34** .pp67–73, 1988. doi:10.1016/S0004-9514(14)60600-0.

[21] P. Kumar, A. Prasad and R. Ghosh, "Stable Phase-Locking of an External-Cavity Diode laser Subjected to External Optical Injection", *J. Phys. B At. Mol. Opt. Phys.* **41**, 2008. doi:10.1088/0953-4075/41/13/135402.

[22] P. Kumar and F. Grillot, "Control of Dynamical Instability in Semiconductor Quantum Nanostructures Diode Lasers: Role of Phase-Amplitude Coupling", *Eur. Phys. J. Spec. Top.* **222** .pp 813–20, 2013. doi:10.1140/epjst/e2013-01884-1.

[23] U.D.E. Les, I. Balears and D.D.E Fisica, "Nonlinear Dynamics of Mutually Coupled Semiconductor Lasers", 2004.

[24] Xia G.Q., Chan S.C and J.M. Liu, "Multistability in a Semiconductor Laser with Optoelectronic Feedback", *Opt Express.* **15** .pp 572–76, 2007. doi:10.1364/OE.15.000572.

[25] Tang S., Vicente R, M.C. Chiang, C.R. Mirasso, and J.M. Liu. "Nonlinear Dynamics of Semiconductor Lasers with Mutual Optoelectronic Coupling", *IEEE J. Sel. Top. Quantum Electron.* **10** PP 936–43, 2004. doi:10.1109/JSTQE.2004.837013.