Initial Transmission Influence on Saturable Absorber Absorption Activity of Passive Q-Switching Erbium-Doped Fiber Laser System

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Abstract—The initial transmission effect on the activity of Cr$^{+3}$: YAG saturable absorber with Er$^{+3}$ doped fiber laser was investigated. Software computer program was built in this study for solving the rate equations model by Runge - Kutta – Fehlberg numerical method. The study reported that the absorption activity of ground state of saturable absorber decreases with the decreasing of the initial transmission, while the activity absorption of excited state begins to increase. Moreover, the total absorption activity of saturable absorber also decreases with the decreasing of the initial transmission.

Keywords— Passive Q-Switched, Er$^{+3}$ Doped Fiber Laser, absorption activity.

I. INTRODUCTION

Passively Q-switched fiber lasers (PQFL) have been widely applied in material processing, range findings, telecommunications, medicine, spectroscopy and biomedical research and micro imaging [1-4]. PQFL have more the attention owing to their advantages of compactness, low cost, mechanical stability, and simple configurations [5,6] . The saturable absorber (SA) is a key element for the passively Q-switched fiber laser [7,8]. Erbium (Er$^{3+}$) doped materials have attracted a lot of attention, because of their potential applications in optoelectronics, in its trivalent state, the Er$^{3+}$ ion shows an intra 4 f shell transition from its first excited state ($^4I_{13/2}$) to the ground state ($^4I_{15/2}$) as shown in Figure(1) [9]. Cr$^{4+}$ doped crystals have wide using in SA preparation such as Cr$^{4+}$: YAG, Cr$^{4+}$: GSGG, Cr$^{4+}$: YSO etc. These crystals have a large absorption cross section and low saturable intensity at the laser wavelength [10, 11]. Cr$^{4+}$:YAG crystal is widely adopted as saturable absorber for passively Q-switched laser generation [12,13]. For a range of laser media, Cr$^{4+}$: YAG has also been successfully used as passive Q-switches, and its energy state can be illustrated by Figure (2) [14]. The absorption activity of SA plays an important role in determining the efficiency of laser optical systems which are used in the generation of high power pulses. This research affects the initial transmission on the absorption activity of SA, especially the emission spectrum for erbium-doped fiber laser system.

![Energy state of Er$^{+3}$](image)

II. THEORY

Coupled rate equations model [15] has been was used in this study for the investigation the effect of SA initial transmission on the absorption activity of SA in Passive Q-switching Er$^{+3}$ doped fiber laser system using as the following equations:

\[
\frac{d\phi(t)}{dt} = \frac{\phi(t)}{\tau_r}[-2\sigma_{l\to g}N(t) - 2\sigma_{l\to g}N(t) - 2\sigma_{l\to g}N(t) - (ln(1 - \frac{1}{R}) + L_{sat})] \quad (1)
\]
\[
\frac{dN(t)}{dt} = R_p - \gamma_c \sigma_{am} \phi(t) N(t) - \frac{N(t)}{\tau_{am}} \tag{2}
\]

Figure (2) : Energy state of Cr⁺ : YAG [14]

\[
\frac{dn_{gs}(t)}{dt} = \frac{n_{gs}(t)}{\tau_{gs}} - 2\sigma_{gs} l_{sa} \phi(t) n_{gs}(t) / \tau_r, \tag{3}
\]

\[
\frac{dn_{es}(t)}{dt} = -\frac{n_{es}(t)}{\tau_{es}} + 2\sigma_{es} l_{sa} \phi(t) n_{es}(t) / \tau_r. \tag{4}
\]

Where: \( \phi \) (cm\(^3\)) is the photons number density, 
\( \tau_r = 2l_{am} / c \) (s) is the transit time for one round trip, 
\( l_r \) (cm) is the length of optical cavity, 
\( \sigma_{am} \) (cm\(^2\)) is the active medium emission cross section, 
\( c \) (ms\(^{-1}\)) is the light speed, 
\( \sigma_{gs} \) (cm\(^3\)) is the absorption cross section of SA ground-state, 
\( l_{am} \) (cm) is the length of AM, 
\( l_s \) (cm) is the length of SA, 
\( n_{gs} \) (cm\(^3\)) is the SA ground state population, 
\( N \) (cm\(^3\)) is the active medium population inversion density, 
\( n_{es} \) (cm\(^3\)) is the SA exited state population, 
\( R = (R_1 R_2)^{1/2} \) is the geometric mean of the cavity, 
\( R_1, R_2 \) is the reflectivity of mirrors, 
\( l_{loss} \) is the dissipative optical losses for round trip, 
\( N \) (cm\(^3\)) is the population inversion density, 
\( \sigma_{es} \) (cm\(^2\)) is the absorption cross section of SA exited-state, 
\( \gamma \) is the population reduction factor equal 1, 2 for 4 states and 3 state of active medium system respectively, 
\( R_p \) is the optical pumping rate, 
\( \tau_{sa} \) (s) is the lifetime of the excited state of SA, 
\( \tau_{am} \) (s) is the fluorescence lifetime of the upper laser state.

Compared to the fluorescence life of the upper laser state, SA’s lifetime in (microsecond), [16] with the Q-switched laser pulses, normally have a very short build-up time so that the spontaneous decay in AM and SA can be neglect also the pumping rate during pulse generation is very long capering Q-switched laser pulses build-up time

[17,18]; then Eq.(2), Eq.(3), and Eq.(4) can be reformulated as the below respectively:

\[
\frac{dN(t)}{dt} = -\gamma_c \sigma_{am} \phi(t) N(t) \tag{5}
\]

\[
\frac{dn_{gs}(t)}{dt} = -2\sigma_{gs} l_{sa} \phi(t) n_{gs}(t) / \tau_r \tag{6}
\]

\[
\frac{dn_{es}(t)}{dt} = 2\sigma_{es} l_{sa} \phi(t) n_{es}(t) / \tau_r. \tag{7}
\]

The density number of photons inside the optical cavity is minimum at the initial time, also most of SA molecules are in the ground state (\( n_{gs} \)), then can be regards

\[
n_{gs} \approx n_{so} \text{, } n_{es} \approx 0 \text{, where } (n_{so} = n_{gs} + n_{es}) \text{ is the total number of SA molecules.}
\]

The SA absorption activity is also very high at the initial time from Eq.(1) (\( d\phi / dt \approx 0 \)) can be considered zero , while cannot consider \( \phi(t) = 0 \) cannot be considered equal to zero. Then;

\[
2\sigma_{am} l_{am} N_o - 2\sigma_{gs} l_{sa} n_{so} - (\ln(1 / R) + L_{loss}) = 0 \tag{8}
\]

When the pulse passes through the SA, then the spatial variation of the pulse energy per unit area (E) at any point of the length of SA (at the coordinate along the longitudinal direction of SA) can be expression by [19] :

\[
\frac{dE}{dz} = -h\nu n_{so} (1 - \frac{\sigma_{ew}}{\sigma_{gs}}) [1 - \exp(-\frac{\sigma_{gs} E}{h\nu})] - n_{so} \sigma_{es} \tag{9}
\]

At small energy, the transmission of SA is called small-signal transmission or initial transmission (\( T_0 \)), at this situation can be regards \( \exp(-\frac{\sigma_{gs} E}{h\nu}) \approx (1 - \sigma_{gs} E / h\nu) \)

and substituted into Eq. (9), get:

\[
\frac{dE}{dz} = [n_{so} \sigma_{gs} - n_{so} \sigma_{es} - n_{so} \sigma_{es}] E
\]

Given that \( \sigma_{gs} \) greater than \( \sigma_{es} \). Then, we can neglect the term that \( \sigma_{es} \):

\[
\ln E_{E_{max}} = n_{so} \sigma_{gs} \int_0^{l_{sa}} d\zeta
\]

The optimization of \( E_{max} \) occurs when the SA bleaches allowing maximum transmission of photons, then we can estimate \( E_{max} \approx \phi_{max} h\nu \). While the optimization of \( E_{min} \)
occurs when the SA is at the high absorption activity, or at small signal transmission of photons, then we can estimate 
\[ E_{\text{min}} \approx T_{\text{opt}} \phi_{\text{max}} h \nu. \]

\[
\ln \frac{E_{\text{max}}}{E_{\text{min}}} = \ln \frac{\phi_{\text{max}} h \nu}{T_{\text{opt}} \phi_{\text{max}} h \nu} = \ln \left( \frac{1}{T_{\text{opt}}} \right) = n_{so} \sigma_g \ell_s a
\]

\[ T_{\text{opt}} = \exp \left(-n_{so} \sigma_g \ell_s a\right) \quad (10) \]

\[
\ln \left( \frac{1}{T_{\text{opt}}} \right) = 2n_{so} \sigma_g \ell_s a
\]

Substituted Eq.(11) into Eq. (8), getting :

\[ N_{\text{opt}} = \frac{\ln \left( \frac{1}{T_{\text{opt}}} \right) + \ln \left( \frac{1}{R} \right) + L_{\text{loss}}}{2\sigma_{\text{am}} \ell_{\text{am}}} \quad \text{(12)} \]

Eq.(12) represents the initial value of population inversion density \((N_{\text{opt}})\) in term of \(T_{\text{opt}}\). In Eq. (1), the initial time of pulse can be considered as \(N(t) \approx \text{Loss}(t)\), and \(\frac{d\phi}{dt} \approx 0\), then we can rewrite it as:

\[ \text{Loss}(t) = [2\sigma_g, \ell_s N_{gs}(t) + 2\sigma_g, \ell_s N_{es}(t) + (\ln \left( \frac{1}{R} \right) + \text{Loss})] / (2\sigma_{\text{am}} \ell_{\text{am}}) \quad \text{(13)} \]

The first term of eq. (13) represents photons loss due to the ground state absorption activity(Gact), while the second term represents photons loss due to the excited state absorption activity(Eact).

\[ \text{Gact}(t) = 2\sigma_g, \ell_s N_{gs}(t) / (2\sigma_{\text{am}} \ell_{\text{am}}) \quad \text{(14)} \]

\[ \text{Eact}(t) = 2\sigma_{es}, \ell_{sa} N_{es}(t) / (2\sigma_{\text{am}} \ell_{\text{am}}) \quad \text{(15)} \]

At maximum of \(\phi\), in Eq.(1) we can consider \(\frac{d\phi}{dt} \approx 0\), \(n_{es} \approx n_{so}\), that mean \(n_{gs}\) can be neglected. Therefore, we can estimate the threshold population inversion density as:

\[ N_{\text{th}} = \frac{2\sigma_{es} n_{so} \ell_{sa} + \ln \left( \frac{1}{R} \right) + L_{\text{loss}}}{2\sigma_{\text{am}} \ell_{\text{am}}} \quad \text{(16)} \]

The ratio \(\frac{\sigma_{es}}{\sigma_{gs}} = \beta\), then we can rewrite Eq.16 in term of the initial transmission \((T_{\text{opt}})\) as the following :

\[ N_{\text{th}} = \frac{\beta \ln \left( \frac{1}{T_{\text{opt}}} \right) + \ln \left( \frac{1}{R} \right) + L_{\text{loss}}}{2\sigma_{\text{am}} \ell_{\text{am}}} \quad \text{(17)} \]

The instantaneous transmission (time transmission) of a saturable absorber is expressed as following \(s\) [20,21]

\[ T(t) = \exp \left[ -\sigma_g(n_e - n_n(t = 0)) \exp \left( -\frac{t}{\tau} \right) - \sigma_e n_e(t = 0) \exp \left( -\frac{t}{\tau} \right) \right] \quad \text{(18)} \]

The first and second term on the right side represent the instantaneous transmission of ground and excited state of SA, respectively.

### III. RESULTS AND DISCUSSION

The set of rete equations (1, 5-7) were as solved numerically by software computer program that was preparing in this study using Runge Kutta –Fehlberg method. The data was used in this study reported as in table (1):

**TABLE (1): THE INPUT DATA**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Ref.</th>
<th>Parameter</th>
<th>Ref.</th>
<th>Parameter</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\ell_{\text{am}} = 25) cm</td>
<td>[22]</td>
<td>(R_1 = 90)%</td>
<td>(\sigma_{es} = 2.25 \times 10^{-14}) cm(^2) s</td>
<td>[23]</td>
<td></td>
</tr>
<tr>
<td>(\tau_\text{m} = 0.575 \times 10^{-14}) cm(^2) s</td>
<td>[22]</td>
<td>(\tau_{es} = 4 \times 10^{-14}) s</td>
<td>(\lambda = 30) cm</td>
<td>[22]</td>
<td></td>
</tr>
<tr>
<td>(\tau_{nm} = 5.545 \times 10^{-13}) s</td>
<td>[22]</td>
<td>(\tau_{en} = 8.75 \times 10^{-14}) cm(^2) s</td>
<td>(\lambda = 1480) nm</td>
<td>[23]</td>
<td></td>
</tr>
</tbody>
</table>

Figure (3) shows the relationship between the time when the optical bleaching state of the SA occurs and the time when the minimum value of difference between the population of the excited state and the ground state of SA, it shows the residual transmission starts from the optical bleaching time (532 ns) to the pulse disappearance time (752 ns). This study explains that there are two important factors in determining the instantaneous transmission and the absorption activity of SA. The first factor is the numerical density of the ions that occupy each of the ground and excited laser states. The second factor is the absorption cross section value of both states. It was observed from the calculations that the lowest value of the difference between the ions density number of the excited laser state and the ground state of SA occurs approximately at 516 ns. The ground laser state population was \(1.495 \times 10^{19}\) ions per
unit volume, and the excited laser state was \(1.518 \times 10^{19}\) ions per unit volume. The reason why the optical bleaching state did not occur at the 516 ns, was due to the second factor. The absorption cross section value of the excited state was less than the absorption cross section of the ground state of SA. To satisfy the difference in activity, it is necessary to increase the population in the excited laser state. This increment was satisfied after 16 ns approximately, and then the optical bleaching time approached to 532ns (516+16). Figure (4) shows the effect of \(T_o\) at value% 0.137, where we note that the absorption activity of the excited state (absorption photons) is significantly lower than that of the ground state until after time 516 ns, although it is more population than the ground state as shown in figure (3). The figure also shows that the final value of the total absorption activity (saturation value) is \(2.1853 \times 10^{19}\).

Figures (5) represents the case of \(T_o=0.117\%\). In Fig.5, we can notice that the physical behavior and interpretation similar to the behavior and its interpretation of in Fig (3). The difference between the two figures (3&5) is clear in the values and behavior of time. Figure (5) shows the lowest value of the density number difference between the ions of the excited and the ground states of SA that is required for optical bleaching taking place (at 444 ns seemed ). This is also occur at advanced time than the of the optical bleaching of state when \(T_o=0.137\%\). Figure (6) shows the effect of \(T_o\) at value% 0.117 on the photons absorption and the instantaneous transmission. It is seemed that the absorption activity of the excited state (absorption photons) is slightly lower than the absorption activity of the ground state even after the 444 ns.. However the population of excited state is more than the ground state as shown in figure (5). Figure 6 also shows that the saturated value of the total absorption activity (saturation value) is \(2.0312 \times 10^{19}\) which is lower than the case at \(T_o = 0.137\%\). Despite that we can notice increasing in the absorption activity of the excited state caused by the decrease in the absorption activity of the ground state.

Figure (4) : The photons absorption and the instantaneous transmission of photons as a function of time at \(T_o=0.137\%\)

Figure (5) : Synchronization of optical bleaching time with smaller difference of SA states at \(T_o=0.117\%\)

Figure (7) represents the case of \(T_o=0.097\%\) enhance the results in previous figures of (\(T_o=0.137\%, T_o=0.117\%\)). We can notice that the physical behavior and interpretation in figure 7 is similar to the behavior and interpretation in the previous figure. Figure (7) shows the lowest value of the difference between the ions density number of the excited
and the ground state of the SA that are required for optical bleaching taking place (at 412 ns approximately). This is also advance time than the cases at $T_o=0.137\%$ and $T_o=0.117\%$. This study explains that is due to the optical bleaching state occurring in advanced time whenever the value of the $T_o$ decreases the decreasing in the value of $T_o$ means that there is high absorption activity led to the rapid translation of the ground state ions to excited state of SA, and causing the optical bleaching state occurs in advanced time.

![Figure 6](image6.png)

Figure 6: The photons absorption and the instantaneous transmission of photons as a function of time at $T_o=0.117\%$

![Figure 7](image7.png)

Figure 7: Synchronization of optical bleaching time with the smaller difference of SA states at $T_o=0.097\%$

![Figure 8](image8.png)

Figure 8: The photons absorption and the instantaneous transmission of photons as a function of time at $T_o=0.097\%$

Figure (8) shows the effect of $T_o$ at 0.097. It seemed that the absorption activity of the excited state of SA (absorbent photons) is slightly greater than the absorbent activity of the ground state of SA after the 412 ns moment due to the significant increase in its population compared to the ground state as shown in figure (7). That is more dominator of the absorption cross section of ground state. The figure also shows that the saturation value of total absorptive activity is $1.8997E+19$, and it is lower than the case of $T_o = 0.117\%$. Although, there is increasing in the absorptive activity of the excited state, due to the high decreasing in the absorption activity of the ground state.

IV. CONCLUSIONS

From this study, we conclude that the activity absorption of saturable absorber ground state decreases with the decreasing of the initial transmission, while the activity absorption of excited state begins to increase. The total absorption activity of SA also decreases with decreasing of the initial transmission.

CONFLICT OF INTEREST

Authors declare that they have no conflict of interest.

V. REFERENCE


